

SHUFE, Fall 2013
Intermediate Macroeconomics
Professor Hui He

Homework 4 Suggested Answer (Total: 105 points)

Due on November 28, Thursday

Endogenous Growth Model

Question 2, 3, 4 on page 247-248 (4th edition: no. 3, 4, 5 on page 258)

Question 2 on page 247 (10 points)

Answer:

An increase in the marginal product of efficiency unit of labor increases the real wage rate, and increases output. However, the increase in z does not change the equilibrium growth rates. The economy has higher paths for consumption and output, but the two paths share the same growth rate.

Question 3 on page 247-248 (10 points)

Answer:

(a) The equation of motion for the economy is now given by:

$$H' = b(1 - u - v)H$$

A change in v , holding u constant, has no effect on the path of H . Consumption is lower because the time spent working for the government cannot produce consumable goods. The two paths of $\log C$ are depicted in the top panel of Figure 7.1

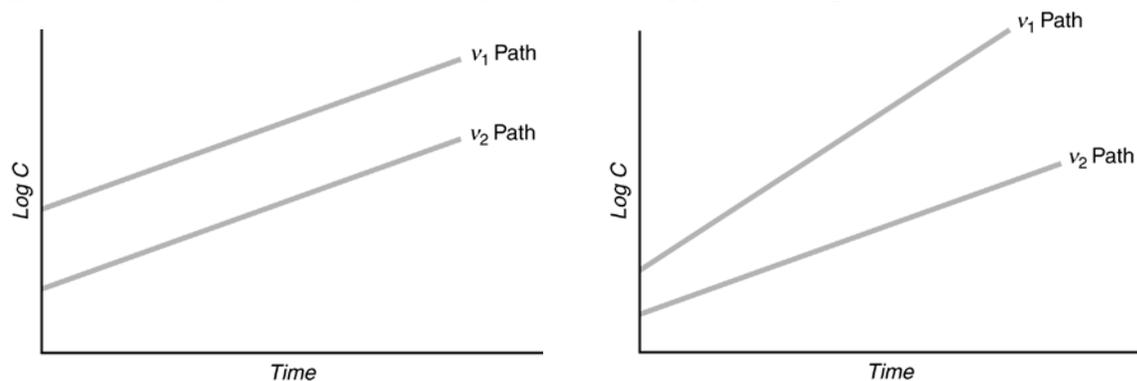


Figure 7.1

(b) Holding u constant, an increase in v reduces the growth rate of human capital. The level of consumption falls as workers are taken away from producing consumption goods. The growth rate of consumption also decreases due the reduction of the growth rate of human capital. The two consumption paths are depicted in the lower panel of Figure 7.1. Offsetting changes in u and v change the level of consumption. However, the equation of motion for H is unchanged, so the rate of growth is unchanged. In part c, the growth rate of H is changed, and so the growth rate of C .

Question 4 on page 248 (10 points)

Answer:

The one-time expenditure lowers the growth path of consumption with no change in the growth rate. The increase in b increases the growth rate of the economy. In the short run, the economy gets less consumption. In the long run, the new growth path eventually surpasses the original growth path. Whether such an investment is worthwhile depends on consumers' preferences for current as opposed to future consumption.

“Working with the data” question 2 on page 249 (10 points)

Answer:

Type	1960 y	1995 y	growth rate (%)
High	4251.41	7040.99	65.62%
Middle	14156.97	27562.59	94.69%
Low	25901.36	46767.91	80.56%

The data does not indicate the tendency for convergence among these three types of countries because the low GDP countries actually have the lowest growth rate between 1960 and 1995.

A Two-Period Model: The Consumption-Savings Decision

Question 1, 2, 4, 5, 8 on page 306-307 (4th edition: question 1, 2, 4, 5, 8 on page 310-311)

Question 1 on page 306 (20 points)

Answer:

Given Information:

$$y = 100$$

$$y' = 120$$

$$t = 20$$

$$t' = 10$$

$$r = 0.1$$

(a) To calculate wealth, we compute:

$$w = y - t + \frac{y' - t'}{1 + r} = 80 + \frac{110}{1.1} = 180$$

(b) In the perfect complements case, the indifference curve are like I_1 and I_2 in Figure 8.1.

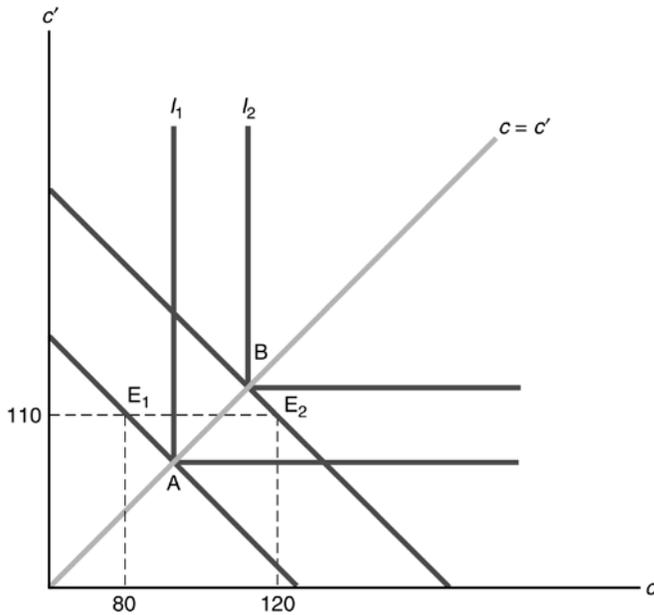


Figure 8.1

(c) The consumer's optimal consumption bundle is at point A. Point A simultaneously solves:

$$c = c', \text{ and}$$

$$c + \frac{c'}{1+r} = c + .91c' = 180$$

Upon solving, we find that $c = c' = 94.2$. Savings is therefore given by:
 $s = y - t - c = 80 - 94.2 = -14.2$

The consumer is a borrower. In Figure 8.1, the endowment point is E_1 and the consumer chooses A.

(d) First-period income rises from 100 to 140. We now recomputed $w = 220$. Solving as in part c, we find that $c = c' = 115.2$, and $s = 4.8$. In Figure 8.1, the endowment point is E_2 and the consumer chooses B.

(e) In part c, the consumer is a borrower. In part d, first-period income increases and savings has consequently increased enough that the consumer is now a lender.

Question 2 on page 306 (10 points)

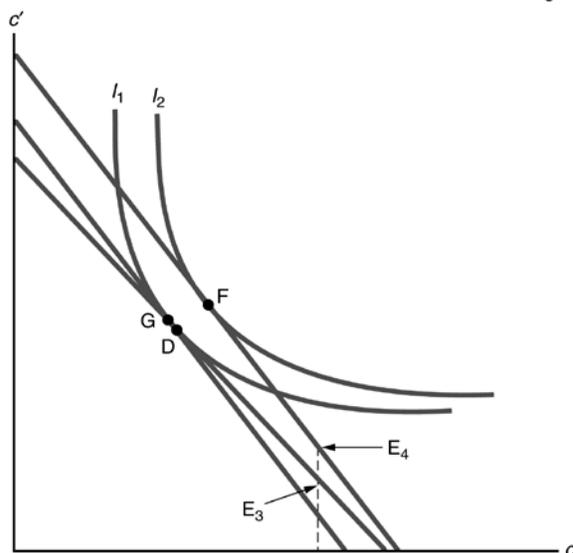
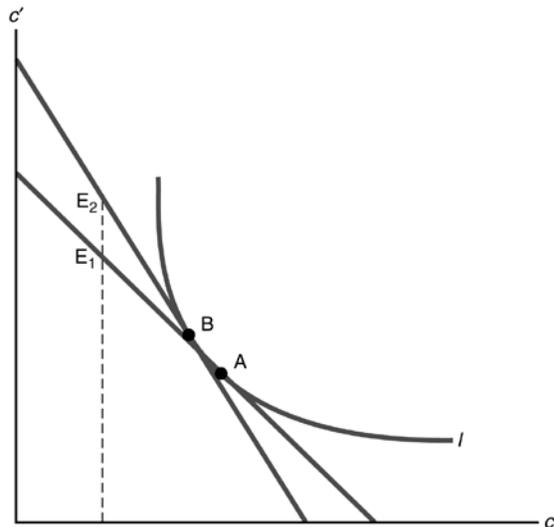
Answer:

In this problem, there is a simultaneous increase in both future income and the real interest rate. The increase in future income is a positive income effect for both borrowers and lenders. The increase in the real interest rate includes a pure substitution effect and a pure income effect. The substitution effect induces the consumer to consume less in the current period and more in the second period. The direction of the pure income effect part of the real interest rate change depends on whether the consumer is a borrower or a lender. Lenders are better off with a higher real interest rate and borrowers are worse off with a higher real interest rate.

The top figure below shows the case of a borrower. The consumer starts out with endowment E_1 and picks point A on indifference curve I . The diagram shows the case in

which the positive income effect of the increase in y is exactly canceled out by the negative income effect of the increase in r . In this particular case, c falls and c' increases. Since current income is fixed, the consumer must increase saving. For the borrower, this amounts to a reduction in borrowing. The consumer therefore picks point B, which is also on indifference curve I , but which is parallel to a budget line that passes through E_2 .

The bottom figure below shows the case of a lender. The consumer starts out with endowment E_3 . The consumer chooses point D that is a tangency of indifference curve I_1 with the budget line that passes through point E_3 . The disturbance shifts the budget line out to the line that passes through E_4 , the new endowment point. The substitution effect moves the consumer from point D to point G on I_1 . The pure substitution effect induces a reduction in c and an increase in c' . The net income effect is then represented by a parallel shift in the line through G to the new budget line. In this case, the two income effects move in the same direction. Therefore both c and c' increase from point G to point F. Second-period consumption unambiguously increases. First-period consumption (and therefore savings) may either rise or fall. The bottom figure below shows the case in which c increases. If c increases, s must fall.



Question 4 on page 306 (10 points)

Answer:

Temporary and Permanent Tax Increases.

(a) The increase in first-period taxes induces a parallel leftward shift in the budget line. The original budget line passes through the initial endowment, E_1 . The new budget line passes through E_2 . The consumer reduces both current and future consumption. In Figure 8.4 the consumer's optimum point moves from point A to point B. First-period consumption falls by less than the increase in taxes and so savings falls.

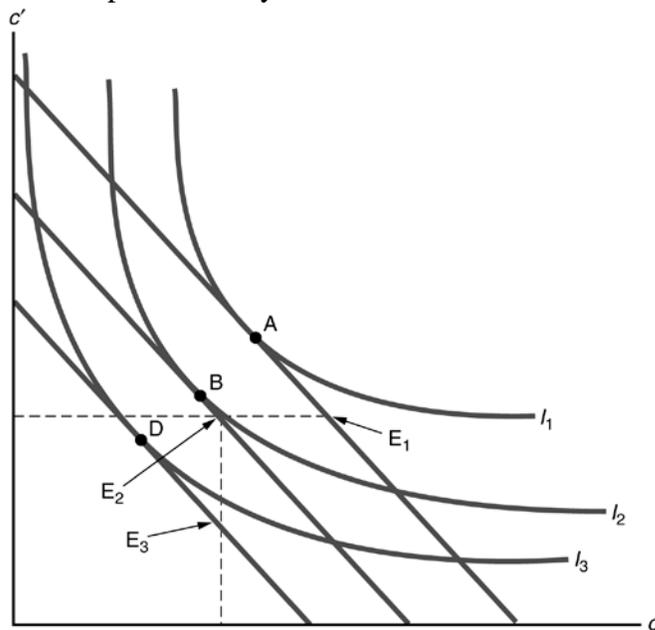


Figure 8.4

(b) Next consider a permanent increase in taxes. A permanent tax increase adds a second tax increase to the first tax increase, the current-period tax increase. The increase in second-period taxes induces a parallel downward shift in the budget line. The new budget line passes through E_2 in Figure 8.4. The second part of the tax increase also reduces both first-period and second-period consumption. The consumer moves from point B to point D. Because the second tax increase reduces first-period consumption holding first-period disposable income fixed, savings must rise. Since the permanent tax increase is the sum of the two individual tax increases, the permanent tax increase reduces both first-period and second-period consumption, but on net, savings may either rise, fall, or remain unchanged.

Question 5 on page 307 (10 points)

Answer:

A tax on interest income.

(a) Initially, AB in Figure 8.5 depicts the consumer's budget constraint. The introduction of the tax results in a kink in the budget constraint, since the interest rate at which the consumer can lend, $r(1-t)$, is now smaller than the interest rate at which the consumer borrows, r . The kink occurs at the endowment, E.

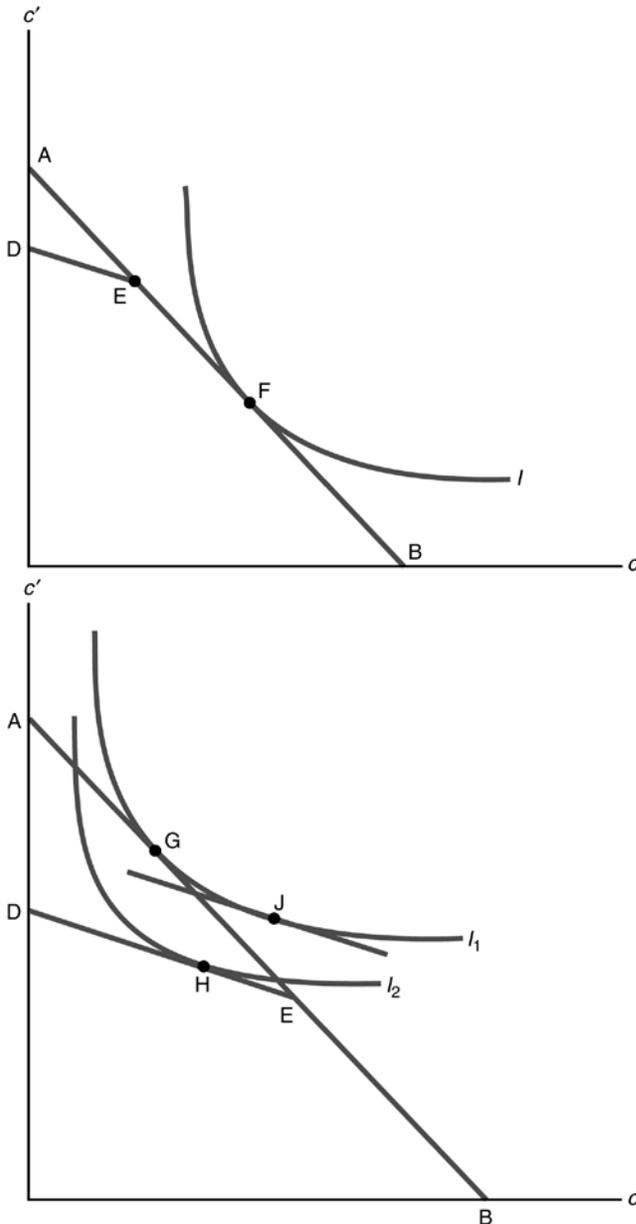


Figure 8.5

(b) The top panel of Figure 8.5 shows the case of a consumer who was a borrower before the imposition of the tax. This consumer is unaffected by the introduction of the tax. The bottom panel of Figure 8.5 shows the case of a consumer who was a lender before the imposition of the tax. Initially the consumer chooses point G, and then chooses

point H after the imposition of the tax. There is a substitution effect that results in an increase in first-period consumption and a reduction in second-period consumption, and moves the consumer from point G to point J. Savings also falls from point G to point J. The income effect is the movement from point J to point H, and the income effect reduces both first-period and second-period consumption, and increases savings. On net, consumption must fall in period 2, but in period 1, consumption may rise or fall. Figure 8.5 shows the case in which first-period consumption increases, which is a case where the substitution effect dominates.

Question 8 on page 307 (15 points)

Answer:

Given information:

$$y = 200$$

$$y' = 150$$

$$t = 40$$

$$t' = 50$$

$$r = 0.05$$

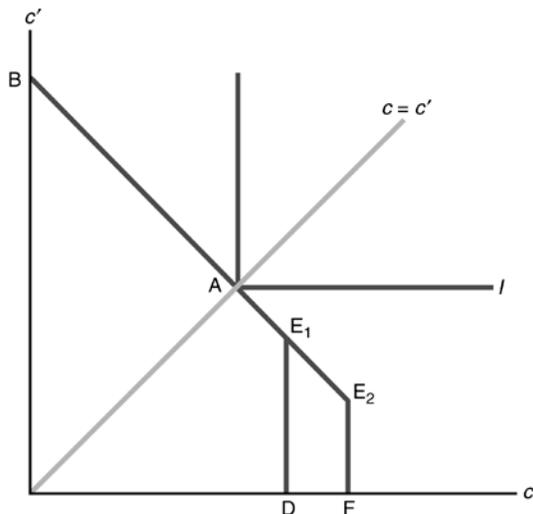
(a) If the consumer could borrow and lend at the real interest rate, $r = 0.05$, then the consumer's lifetime budget constraint would be given by:

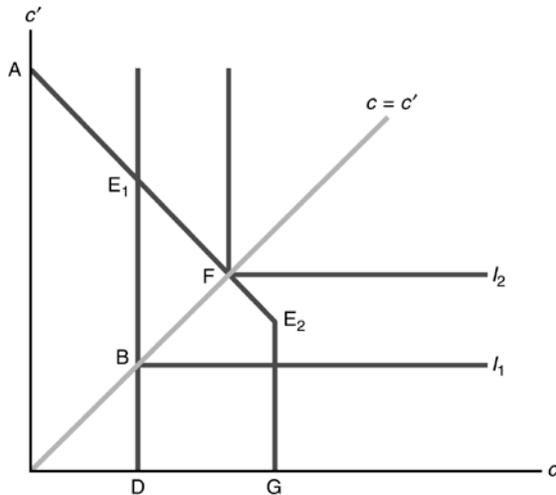
$$c + \frac{c'}{(1+r)} = y - t + \frac{y' - t'}{(1+r)}.$$

Plugging in the numbers from this problem, we obtain:

$$c + 0.95c' = 255.2.$$

In the figure below, the initial budget constraint is given by BE_1D . The budget constraint has a kink at the initial endowment point $E_1 = (160, 100)$, because the consumer cannot borrow, and therefore cannot consume more than 160 in the first period. Because the consumer has perfect-complements preferences, the indifference curves are kinked at $c = c'$.





(b) With perfect-complements preferences, the consumer picks point A in figure on the previous page. Plugging in $c = c'$ into the budget constraint and solving, we find that $c = c' = 130.7$ and so $s = y - t - c = 160 - 130.7 = 29.3$. In this case, the fact that the consumer cannot borrow does not matter for the consumer's choice, as the consumer decides to be a lender.

(c) When $t = 20$ and $t' = 71$, the consumer's lifetime wealth remains unchanged at 255.2. However, the budget constraint shifts to BE_2F , figure on the previous page, with the new endowment point at $E_2 = (180, 79)$. This change does not matter for the consumer's choice, again because he or she chooses to be a lender. Consumption is still 130.7, but now savings is $s = y - t - c = 180 - 130.7 = 49.3$.

(d) Now first-period income falls to 100. Wealth is now equal to $w = 155.2$. In the figure above, the budget constraint for the consumer is AE_1D , so when the consumer chooses the point on his or her budget constraint that is on the highest indifference curve, any point on the line segment BE_1 will do. Suppose that the consumer chooses the endowment point E_1 , where $c = 60$ and $c' = 100$. This implies that $s = 0$, and the consumer is credit-constrained in that he or she would like to borrow, but cannot. Now with the tax change, the budget constraint shifts to AE_2G , with the endowment point $E_2 = (80, 79)$. Thus the consumer can choose $c = c'$ on the new budget constraint, and solving for consumption in each period using the budget constraint

$$c + 0.95c' = 155.2,$$

we get $c = c' = 79.5$, and $s = 0.5$. Here, notice that first-period consumption increased by almost the same amount as the tax cut, although lifetime wealth remains unchanged at 155.2. Effectively, the budget constraint for the consumer is relaxed. Therefore, for tax cuts that leave lifetime wealth unchanged, lenders will not change their current consumption, but credit-constrained borrowers will increase current consumption.