European Economic Review 56 (2012) 1546–1572

Contents lists available at SciVerse ScienceDirect

European Economic Review

journal homepage: www.elsevier.com/locate/eer

What drives the skill premium: Technological change or demographic variation?

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ARTICLE INFO

Article history:
Received 29 December 2011
Accepted 10 September 2012
Available online 23 September 2012

JEL classification:
E24
E25
J24
J31
O33
I21

Keywords:
Skill premium
Schooling choice
Capital–skill complementarity
Investment-specific technological change
Demographic change

ABSTRACT

This paper quantitatively examines the effects of two exogenous driving forces, investment-specific technological change (ISTC) and the demographic change known as ‘‘the baby boom and the baby bust,’’ on the evolution of the skill premium and the college enrollment rate in the postwar U.S. economy. We develop a general equilibrium overlapping generations model with endogenous discrete schooling choice. The production technology features capital–skill complementarity as in Krusell et al. (2000). ISTC, through capital–skill complementarity, raises the relative demand for skilled labor, while demographic variation affects the skill premium by changing the age structure and hence the relative supply of skilled labor. We find that ISTC is the key element driving the skill premium in the postwar U.S. economy. And it is quantitatively important for the dynamics of the college enrollment rate. The quantitative importance of the demographic change for the evolution of both the skill premium and the college enrollment rate however is limited.

1. Introduction

The *skill premium*, which is defined as the ratio of the wage of skilled labor (workers holding college degrees) to the wage of unskilled labor (workers holding high school diplomas), has gone through dramatic changes in the postwar U.S. economy. As Fig. 1 shows, starting in 1949 the evolution of the skill premium (expressed in natural logarithm) exhibited an ‘‘*N* ’’ shape: it increased in the 1950s and most of the 1960s, then decreased throughout the 1970s, and has increased dramatically since then. Meanwhile, the figure also shows the relative supply of skilled labor (the ratio of annual hours worked by skilled labor to annual hours worked by unskilled labor) has been increasing steadily over time.[1]

A number of researchers have asked why the pattern of the skill premium looks like it does. Popular explanations include investment-specific technological change through capital–skill complementarity (see Krusell et al., 2000, hereafter KORV), skill-biased technological change induced by international trade (Acemoglu, 2003), and skill-biased technological change associated with the computer revolution (Autor et al., 1998). Probably the most popular story is the one proposed

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[1] The annual data since 1963 are taken from the CPS. Data in 1949 and 1959 are obtained from the 1950 and 1960 census. See Appendix A for the details. The pattern of the skill premium in Fig. 1 has been widely documented in the literature. For example, Fig. 1 in Acemoglu (2003) shows a similar pattern for both the skill premium and the relative supply of skilled labor.

0014-2921/$ - see front matter
http://dx.doi.org/10.1016/j.euroecorev.2012.09.005

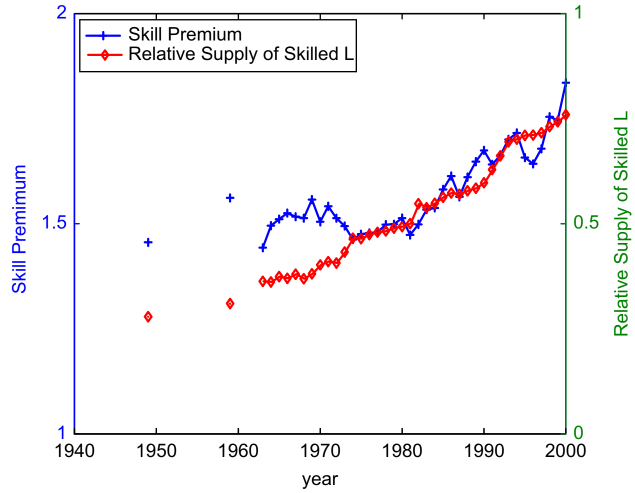

Fig. 1. The skill premium (log units) and relative supply of skilled labor.

in Katz and Murphy (1992). They claim that a simple supply and demand framework with a smooth secular increase in the relative demand for skilled labor combined with the observed fluctuations in the rate of growth of the relative supply of skilled labor can potentially explain the dynamics of the skill premium. They attribute the fluctuations in the relative supply of skilled labor mainly to the demographic change. They also argue that the accelerating skill-biased technological change in the 1980s has contributed to the rising college wage premium since 1980 via raising the relative demand of skilled labor.

This paper contributes to the literature by asking a quantitative question: *to what extent can skill-biased technological change and demographic change account for the dynamics of the skill premium, respectively*? We develop a general equilibrium overlapping generations model with endogenous discrete schooling choice. The model includes three key features. First, with *ex-ante heterogeneity in the disutility cost of schooling*, individuals in each birth cohort (high school graduates) choose to go to college or not based on their expected future wage differentials, their forgone wages during the college years, their tuition payments, and their idiosyncratic disutility cost. This microfoundation gives us the standard features found in the human capital investment literature (see, for instance, Ben-Porath, 1967). Second, the production technology has the feature of *capital–skill complementarity* as in KORV (2000); that is, capital is more complementary to skilled than to unskilled labor. Third, following Greenwood et al. (1997, hereafter GHK), we assume the existence of *investment-specific technological change* (*ISTC*).

In this model, ISTC and demographic change drive the equilibrium outcomes of the skill premium by dynamically affecting the relative demand and supply of skilled labor. ISTC, through the capital–skill complementarity in the production technology, increases the relative demand of skilled labor and thus raises the skill premium. In turn, the rising skill premium encourages skill formation and increases the relative supply of skilled labor. In contrast, demographic change affects the age structure in the economy. A change in the age structure has a direct impact on the relative supply of skilled labor. In addition, since people have different saving tendencies along the life-cycle, a change in the age structure also influences the relative demand for skilled labor through changing asset accumulation in the economy. The ultimate effects of these two forces on the skill premium (and college enrollment rate) depend on the quantitative magnitude of both demand and supply effects.

We calibrate the model to match the U.S. data for the period 1947–1951 as the initial steady state. Then, by feeding in the ISTC data from Cummins and Violante (2002) and the growth rate of the high school graduates cohort size from 1951 to 2000, we conduct perfect foresight deterministic simulations to compare with the data of the 1951–2000 period and counterfactual decomposition experiments to identify the effects of each force. We find that ISTC plays a dominant role in driving the dramatic increase in the skill premium. It alone captures about 82% of the increase in the skill premium for the period 1951–2000, while the quantitative importance of the demographic change to the evolution of the skill premium is limited, especially after 1980. In addition, we find that ISTC can explain about 35% of the increase in the college enrollment rate for the period 1951–2000, while demographic change does not have a significant effect on the college enrollment rate over time.

This paper extends the existing literature on the effects of skill-biased technological change on wage inequalities such as KORV (2000). Compared to KORV (2000), our extension lies in two dimensions. First, we embed ISTC and capital–skill complementarity in a dynamic general equilibrium framework. Second, we endogenize the relative supply of skilled labor by modeling the college attendance choice. Therefore, the model is able to capture the dynamic interaction between the skill premium and the relative supply of skilled labor. With these extensions, the paper can shed light on the impact of two forces on the evolution of both the skill premium and college enrollment rate. As far as we know, this is the first paper to use a general equilibrium model with ISTC and endogenous college entrance decision to analyze the evolution of the skill premium in the U.S.[2]

In spirit, this paper is also close to Heckman et al. (1998). They develop and estimate a general equilibrium overlapping generations model of labor earnings and skill formation with heterogeneous human capital.[3] They test their framework by building into the model a baby boom in entry cohorts and an estimated time trend of increase in the skill bias of aggregate technology. They find that the model can explain the pattern of wage inequality since the early 1960s. However, they do not provide a microfoundation for the source of skill-biased technological change (i.e., ISTC) as in this paper. They also do not ask the research question about the quantitative decomposition of the impacts of ISTC and demographic change on both the skill premium and the college enrollment rate.[4]

This paper extends He and Liu (2008), who provide a unified framework in which the dynamics of the relative supply of skilled labor and the skill premium arise as an equilibrium outcome driven by measured investment-specific technological change. This paper provides a microeconomic foundation for He and Liu (2008) by going deeper into the college choices that determine the supply of skilled labor. An overlapping generations framework is used here to allow for a better model of educational attainment. The model also includes a time-varying cohort size that can be used to quantify the effect of demographic change and allows for a better analysis of labor supply. Both elements are missing in He and Liu (2008).

The remainder of this paper is organized as follows. Section 2 documents some stylized facts about the dynamics of the cohort size of high school graduates, the college enrollment rate, and college tuition in the postwar U.S. economy. Section 3 presents the economic model of the decision to go to college, describes the market environment, and defines the general equilibrium in the model economy. Section 4 shows how to parameterize the model economy. Section 5 provides calibration results for the pre-1951 steady state and shows some comparative static results. Section 6 computes the transition path of the model economy from 1951 to 2000 and compares the results with the data. It also conducts some counterfactual experiments to isolate the effects of ISTC and demographic change on the skill premium and college enrollment rate. Section 7 addresses the importance of endogenizing college choice and discusses the limitation of the benchmark model in replicating the college attendance rate. Section 8 provides several sensitivity analyses. Finally, Section 9 concludes.

2. Stylized facts

This section summarizes the data pattern regarding the college choice that determines the supply of skilled labor. See Appendix A for the source and construction of the data. Fig. 2 shows the cohort size of high school graduates. It was very stable before the early 1950s, then increased until 1976, and has decreased since then. Since the common age of high school graduation is around 18, we can view this graph as an 18-year lag version of U.S. fertility growth; that is, it reflects the baby boom and baby bust.[5]

Fig. 3 measures the college-age population. We report the population of age 18–21 in the U.S. since 1955. These series follow a pattern similar to that in Fig. 2. The baby boom pushed the college-age population up until the fertility rate reached its peak around 1960, corresponding to the peak of the college-age population around 1980. The baby bust then dragged the population size down.

The two figures above show changes in the population base of potential college students, but does the *proportion* of people going to college change over time? Fig. 4 shows the college enrollment rate of recent high school graduates. It began growing in the early 1950s until 1968, when it started to decline; the 1970s were a decade of depressed college enrollment, and it was not until 1985 that the enrollment rate exceeded the level in 1968. Starting in 1980, the enrollment rate kept increasing for nearly 20 years. This pattern is also confirmed by other studies. (See Macunovich, 1996, Figs. 1.a, 1.b, 2.a, and 2.b and Card and Lemieux, 2000, Fig. 3.)

Combining Figs. 3 and 4, we can see that the dramatic increase in the relative supply of skilled labor in the 1970s is due to a combination of a rising college-age population and a rising college enrollment rate in the 1960s. The demographic change affects only the *cohort size* of the college-age population. The higher enrollment rate shifts the proportion of the college-age population into the skilled labor pool. The college enrollment rate thus is an important determinant of the relative supply of skilled labor.

[2] Restuccia and Dandenbroucke (2010) examine the quantitative contribution of changes in the return to schooling, which imply skill-biased technological change, in explaining the evolution of educational attainment in the U.S. from 1940 to 2000 in a similar model with discrete schooling choice. Their model, however, does not include capital accumulation and capital–skill complementarity and hence cannot address the effect of ISTC on the skill premium via physical capital accumulation.

[3] Several other papers follow Heckman et al. (1998) to emphasize the impact of skill-biased technological change on the skill premium. For example, Guvenen and Kuruscu (2006) present a tractable general equilibrium overlapping generations model of human capital accumulation that is consistent with several features of the evolution of the U.S. wage inequality from 1970 to 2000. Their work shares a similar microfoundation of schooling choice as in this paper. But they do not have capital stock in the production technology and, hence, no capital–skill complementarity. The only driving force in their paper is skill-biased technological change, which is calibrated to match the total rise in wage inequality in the U.S. data between 1969 and 1995.

[4] Lee and Wolpin (2010) is a notable exception. Their model, however, does not include asset accumulation. Therefore, it cannot address the interaction between demographic change and asset holdings as we emphasized above.

[5] Cohort size has been increasing since 1995 because the baby boomers' children reached college age around the mid-1990s.

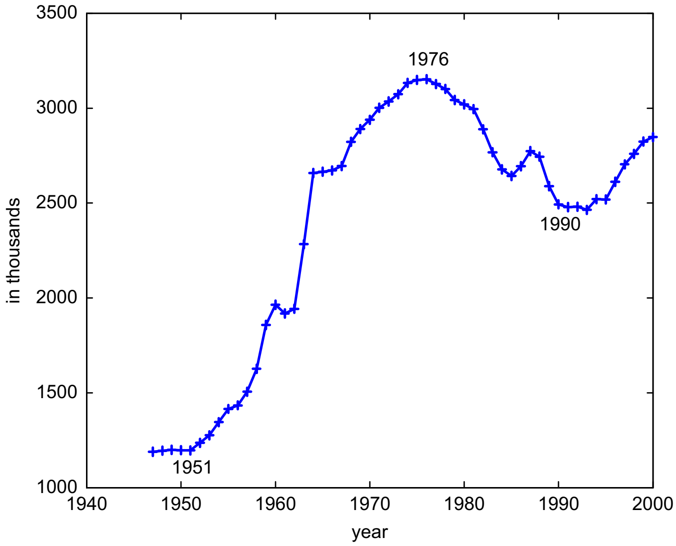

Fig. 2. High school graduates cohort size.

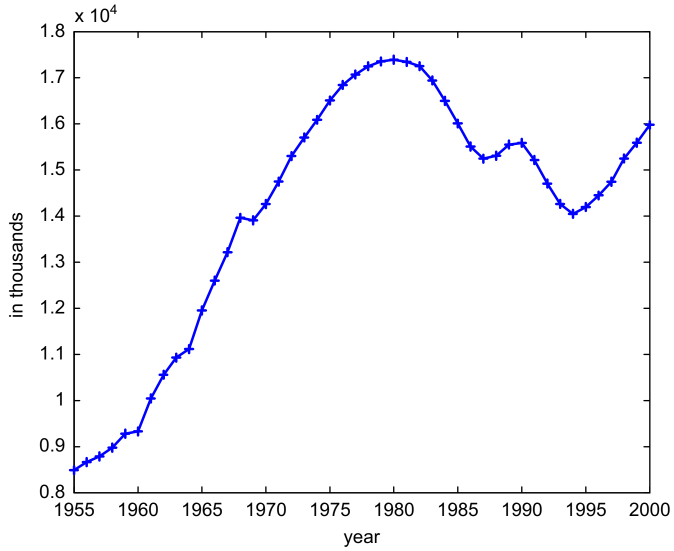

Fig. 3. College age population.

By comparing the skill premium in Fig. 1 and the college enrollment rate in Fig. 4, one can see that they share a very similar pattern. This similarity implies a tight link between the decision to go to college and the expected skill premium. The expected skill premium represents the expected gain from higher education. As the expected benefits increase, the enrollment rate increases. This finding motivates us to explicitly endogenize the college choice in the model. As Goldin and Katz (2007) point out, understanding changes in the relative supply of skilled labor is crucial in the economic analysis of changes in wage structure and returns to skill. In order to understand the evolution of the skill premium, we cannot ignore the impact from the supply side.

To fully understand the determinants of schooling choice, we should also look at the cost side of going to college. In Fig. 5 we report the real tuition, fees, room, and board (TFRB) per student charged by an average four-year institution (average means the enrollment-weighted average of four-year public and private higher education institutions; see Appendix A for details). Again, we see a pattern similar to that of the skill premium and the college enrollment rate. TFRB increased over time except in the 1970s. Starting in 1980, real TFRB have increased dramatically.

The stylized facts relevant to this paper can be briefly summarized as follows:

1. The skill premium rose during the 1950s and 1960s, fell in the 1970s, and has increased dramatically since 1980.
2. The relative supply of skilled labor has increased since the 1940s.
3. The college enrollment rate exhibits a pattern similar to that of the skill premium, as do tuition payments.

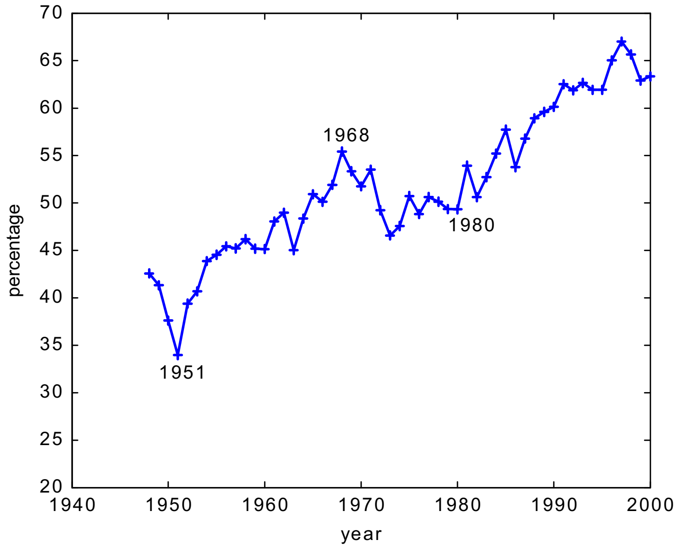

Fig. 4. College enrollment rate of high school graduates.

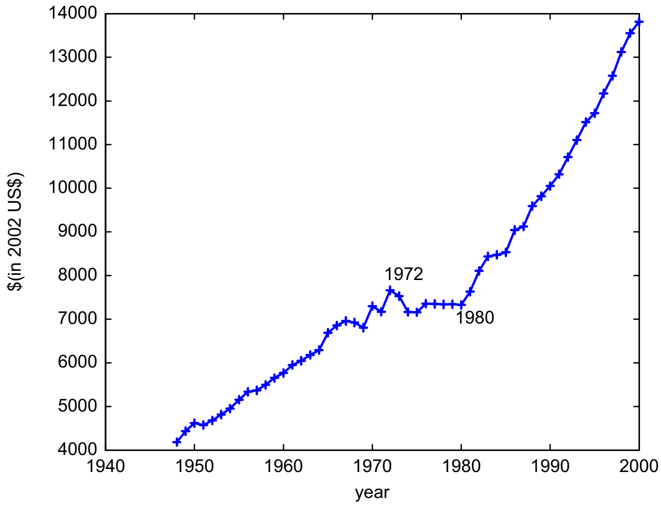

Fig. 5. Average real TFRB charges.

The stylized facts about the skill premium observed in Fig. 1 are the target of this paper. To answer the quantitative question raised in Section 1, we will take the demographic change in Fig. 2 and the measured investment-specific technological change in Fig. 9 as exogenously given, feed them into a dynamic general equilibrium model, and see what percentage of change in the skill premium can be explained by each of these two exogenous forces.

3. Model

In this section, we present the economic model that will be used later for calibration. It is a discrete-time overlapping generations (OLG) model. Individuals make the schooling choice in the first period. There is only one good in the economy that can be used in either consumption or investment.

3.1. Demographics

The economy is populated by overlapping generations. Individuals enter the economy when they are 18 years old and finish high school, which we call the birth cohort and model as age $j=1$. We assume individuals work up to age J, which is the maximum life span. The model period is one year. To distinguish between the age of a cohort and calendar time, we use j for age, and t for calendar time. For example, $N_{j,t}$ is the population size of the age-j cohort at time t.

In every period t a new birth cohort enters the economy with cohort size $N_{1,t}$. It grows at rate n_t. Therefore, we have

$$N_{1,t} = (1+n_t)N_{1,t-1}. \tag{1}$$

The fraction of the age-j cohort in the total population at time t is

$$\eta_{j,t} = \frac{N_{j,t}}{N_t} = \frac{N_{j,t}}{\sum_{i=1}^{J} N_{it}}. \tag{2}$$

This fraction will be used to calculate the aggregate quantities in the economy as cohort weights throughout the transition path.

The birth cohort in the model corresponds to the high school graduates (HSG) in Fig. 2, and the growth rate of the HSG cohort size is the data counterpart of n_t. Therefore, the "baby boom" corresponds to the 1951–1976 period when n_t increased over time, while the "baby bust" period is from 1976 to 1990 when n_t decreased over time.

3.2. Preferences

Individuals born at time t want to maximize their discounted lifetime utility

$$\sum_{j=1}^{J} \beta^{j-1} u(c_{j,t+j-1}).$$

The period utility function is assumed to take the CRRA form

$$u(c_{j,t+j-1}) = \frac{c_{j,t+j-1}^{1-\sigma}}{1-\sigma}. \tag{3}$$

The parameter σ is the coefficient of relative risk aversion; therefore, $1/\sigma$ is the intertemporal elasticity of substitution. Since leisure does not enter into the utility function, each individual will supply all her labor endowment, which is normalized to be one.

3.3. Budget constraints

An individual born at time t chooses whether or not to go to college at the beginning of the first period. We use $s \in \{c,h\}$ to indicate this choice. If an individual chooses $s = h$, she ends up with a high school diploma and goes on the job market to work as an unskilled worker up to age J, and earns high school graduate wage sequence $\{w^h_{j,t+j-1}\}_{j=1}^{J}$. Alternatively, she can choose $s = c$, spend the first four periods in college as a full-time student, and pay the tuition p. We assume that an individual who enters college will successfully graduate from college. After college, she goes on the job market to find a job as a skilled worker and earns a college graduate wage sequence $\{w^c_{j,t+j-1}\}_{j=1}^{J}$. After the schooling choice, within each period, an individual makes consumption and asset accumulation decisions according to her choice. For simplicity, we assume there is no college dropout and no unemployment.

For $s = c$, the budget constraints of the cohort born at time t are

$$c_{j,t+j-1} + p_{t+j-1} + a_{j,t+j-1} \le (1+r_{t+j-1})a_{j-1,t+j-2} \quad \forall j = 1,2,3,4, \tag{4}$$

$$c_{j,t+j-1} + a_{j,t+j-1} \le (1+r_{t+j-1})a_{j-1,t+j-2} + w^c_{t+j-1}\varepsilon^c_j \quad \forall j = 5,\dots,J, \tag{5}$$

$$c_{j,t+j-1} \ge 0,\ a_{0,t-1} = 0, \quad a_{J,t+J-1} \ge 0,$$

where $\{\varepsilon^c_j\}_{j=5}^{J}$ is the age efficiency profile of college graduates. It represents the age profile of the average labor productivity for college graduates. Notice that individuals have zero initial wealth and cannot die in debt.

For $s = h$, the budget constraints of the cohort born at time t are

$$\begin{aligned} &c_{j,t+j-1} + a_{j,t+j-1} \le (1+r_{t+j-1})a_{j-1,t+j-2} + w^h_{t+j-1}\varepsilon^h_j \quad \forall j = 1,\dots,J, \\ &c_{j,t+j-1} \ge 0,\ a_{0,t-1} = 0,\ a_{J,t+J-1} \ge 0. \end{aligned} \tag{6}$$

Similarly, $\{\varepsilon^h_j\}_{j=1}^{J}$ is the age efficiency profile of high school graduates.

3.4. Schooling choice

Next, we would like to explicitly model an individual's schooling choice. In order to generate a positive enrollment rate in the model, we need to introduce some *ex-ante* heterogeneity within each birth cohort. Without this within-cohort heterogeneity, the enrollment rate would be either zero or one.

Following Heckman et al. (1998), we assume that different individuals within each birth cohort are endowed with different levels of the disutility cost of schooling. We index people by their disutility level $i \in [0,1]$, and the associated disutility cost that individual i bears is represented by $\chi(i)$. We assume $\chi'(i) < 0$.[6] The cumulative distribution function (CDF) of the disutility cost is denoted by F, $F(i_0) = \Pr(i \le i_0)$. Now an individual i born at time t has her own expected discounted lifetime utility

$$\sum_{j=1}^{J} \beta^{j-1} u(c_{j,t+j-1}) - \mathcal{I}_i \chi(i), \tag{7}$$

where

$$\mathcal{I}_i = \begin{cases} 1 & \text{if } s_i = c, \\ 0 & \text{if } s_i = h \end{cases}$$

subject to the conditional budget constraints (4), (5) or (6), depending on individual i's schooling choice s_i. Notice that the idiosyncratic disutility cost $\chi(i)$ does not enter into the budget constraints, so everyone within the same cohort and with the same education status will have the same lifetime utility derived from physical consumption, which simplifies the computation. We use V_t^c to denote the discounted lifetime utility derived from consumption for individuals who are born at time t and choose to go to college ($s = c$) and V_t^h to denote the discounted lifetime utility derived from consumption for individuals who choose not to go to college ($s = h$). Therefore, $V_t^c - V_t^h$ represents the utility gain from consumption via attending college. Obviously, individual i will choose to go to college if $\chi(i) < [V_t^c - V_t^h]$, will not go if $\chi(i) > [V_t^c - V_t^h]$, and is indifferent if $\chi(i) = [V_t^c - V_t^h]$.

From this representation it is very clear how the skill premium is going to affect an individual's schooling decision. Keeping other things equal, an increase in the skill premium will raise the benefit of schooling, thus implying a higher utility gain from attending college $V_t^c - V_t^h$. If we assume that the distribution of the disutility cost is stationary, a higher utility gain from schooling means it is more likely that $\chi(i) < [V_t^c - V_t^h]$, which implies that more individuals would like to go to college. This mechanism will generate the co-movement between the skill premium and the enrollment rate as observed in the data.

3.5. Production

We close the model by describing the production side of the economy. The representative firm in the economy uses capital stock (K), skilled labor (S), and unskilled labor (U) to produce a single good. Here skilled labor consists of college graduates, and unskilled workers are high school graduates. Following KORV (2000), we adopt an aggregate production function with capital–skill complementarity as follows[7]:

$$Y_t = A_t F(K_t, S_t, U_t) = A_t[\mu U_t^\theta + (1-\mu)(\lambda K_t{}^\rho + (1-\lambda) S_t^\rho)^{\theta/\rho}]^{1/\theta}, \tag{8}$$

where A_t is the level of total factor productivity (TFP). We also have $0 < \lambda, \mu < 1$, and $\rho, \theta < 1$. This production technology exhibits constant returns to scale. The elasticity of substitution between the capital–skilled labor combination and unskilled labor is $1/(1-\theta)$ and the one between capital and skilled labor is $1/(1-\rho)$. For the capital–skill complementarity, we require

$$\frac{1}{1-\rho} < \frac{1}{1-\theta},$$

which means $\rho < \theta$.

The law of motion for the capital stock in this economy is expressed as

$$K_{t+1} = (1-\delta) K_t + X_t q_t,$$

where X_t denotes capital investment. Following GHK (1997), we interpret q_t as the current state of the technology for producing capital; hence, changes in q represent the notion of investment-specific technological change (ISTC). When q increases, investment becomes increasingly efficient over time.

Defining

$$\tilde{K}_{t+1} \equiv \frac{K_{t+1}}{q_t},$$

it is easy to show that this economy is equivalent to the model with the following production function and capital accumulation:

$$Y_t = A_t[\mu U_t^\theta + (1-\mu)(\lambda (B_t \tilde{K}_t)^\rho + (1-\lambda) S_t^\rho)^{\theta/\rho}]^{1/\theta},$$
$$\tilde{K}_{t+1} = (1-\tilde{\delta}) \tilde{K}_t + X_t,$$

[6] Navarro (2007) finds that ability is the main determinant of this "psychic" cost, and it plays a key role in determining schooling decisions. High-ability individuals face a very low disutility cost, while low-ability individuals face a large disutility cost of attending college. Therefore, we can also view i as the index of individuals' "learning ability."

[7] Griliches (1969) provides evidence from U.S. manufacturing data that skill is more complementary to capital than to unskilled labor. Duffy et al. (2004) show empirical support for the capital–skill complementarity hypothesis by using a macropanel set of 73 countries over the period 1965–1990.

with

$$\tilde{\delta} = 1-(1-\delta)\frac{q_{t-1}}{q_t} \quad \text{and} \quad B_t = q_{t-1}.$$

This transformation maps changes in ISTC into the changes in the capital productivity level B_t.[8] It simplifies the computation of the model. From now on, we refer to this transformed version of the model as the benchmark model.

Based on the transformation, the representative firm's profit maximization implies the first-order conditions as follows:

$$r_t = \lambda(1-\mu)A_t B_t^{\rho} H_t(\lambda(B_t\tilde{K}_t)^{\rho}+(1-\lambda)S_t^{\rho})^{(\theta/\rho)-1}\tilde{K}_t^{\rho-1}-\tilde{\delta}, \tag{9}$$

$$w_t^c = (1-\mu)(1-\lambda)A_t H_t(\lambda(B_t\tilde{K}_t)^{\rho}+(1-\lambda)S_t^{\rho})^{(\theta/\rho)-1}S_t^{\rho-1}, \tag{10}$$

$$w_t^h = \mu A_t H_t U_t^{\theta-1}, \tag{11}$$

where $H_t = [\mu U_t^{\theta}+(1-\mu)(\lambda(B_t\tilde{K}_t)^{\rho}+(1-\lambda)S_t^{\rho})^{\theta/\rho}]^{(1/\theta)-1}$.

Dividing (10) by (11), we derive the expression for the skill premium:

$$\frac{w_t^c}{w_t^h} = \frac{(1-\mu)(1-\lambda)}{\mu}\left[\lambda\left(\frac{B_t\tilde{K}_t}{S_t}\right)^{\rho}+(1-\lambda)\right]^{(\theta-\rho)/\rho}\left[\frac{S_t}{U_t}\right]^{\theta-1}. \tag{12}$$

Log-linearizing (12), and using a "hat" to denote the rate of change between time ($\hat{X} = \Delta X/X$), we obtain (ignoring time subscripts for convenience)

$$\left(\widehat{\frac{w^c}{w^h}}\right) \approx \lambda(\theta-\rho)\left(\frac{B\tilde{K}}{S}\right)^{\rho}[\hat{B}+\hat{K}-\hat{S}]+(\theta-1)[\hat{S}-\hat{U}]. \tag{13}$$

This equation is exactly the same as in KORV (2000) except for the B term. It says that the growth rate of the skill premium is determined by two components. One is the growth rate of the relative supply of skilled labor $[\hat{S}-\hat{U}]$. Since $\theta<1$, relatively faster growth of skilled labor will reduce the skill premium. This term is called the "*relative quantity effect*" in KORV (2000). The other term

$$\lambda(\theta-\rho)\left(\frac{B\tilde{K}}{S}\right)^{\rho}[\hat{B}+\hat{K}-\hat{S}]$$

is called the "*capital–skill complementarity effect*." If capital grows faster than skilled labor, this term will raise the skill premium due to $\rho<\theta$. The dynamics of the skill premium depend on the trade-off between these two effects.

The transformation above maps ISTC q_t into the changes in the capital productivity level B_t. Therefore, increases in q_t will be transformed into increases in B_t. As shown in Eq. (13), when B_t increases, through the capital–skill complementarity effect, it will raise the skill premium. ISTC thus is also skill-biased.

Finally, the resource constraint in the economy is given by

$$C_t+P_t+X_t = Y_t,$$

where C_t is total consumption and P_t is the total tuition payment.

3.6. *The competitive equilibrium*

The model above is a standard OLG setting with discrete schooling choices. We assume that individuals have perfect foresight about the paths of exogenous changes $\{n_t\}$ and $\{q_t\}$.[9] Suppose an individual i born at time t has already made the schooling decision $s_{i,t}$. Conditional on this choice, we can present her utility maximization problem in terms of a dynamic programming representation.

For $s_{i,t}=c$, let $V_{t+j-1}^c(a_{j-1,t+j-2},j)$ denote the value function of an age-j individual with asset holdings $a_{j-1,t+j-2}$ at beginning of time $t+j-1$. It is given as the solution to the dynamic problem

$$V_{t+j-1}^c(a_{j-1,t+j-2},j) = \max_{\{c_{j,t+j-1},a_{j,t+j-1}\}}\{u(c_{j,t+j-1})+\beta V_{t+j}^c(a_{j,t+j-1},j+1)\} \tag{14}$$

subject to (4)–(5).

[8] This transformation is motivated by GHK (1997). See Appendix B in GHK (1997) for a similar transformation for their benchmark economy.

[9] A perfect foresight assumption is quite common in this type of research. McGrattan and Ohanian (2008) use this assumption and conduct deterministic simulations to study the macroeconomic impact of fiscal shocks during World War II. Chen et al. (2006) take the actual time path of the TFP growth rate to investigate its impact on the postwar Japanese saving rate. Their sensitivity analysis shows that alternative expectations hypotheses do not significantly change the quantitative results.

For $s_{i,t}=h$, the corresponding value function is

$$V^h_{t+j-1}(a_{j-1,t+j-2},j)=\max_{\{c_{j,t+j-1},a_{j,t+j-1}\}}\{u(c_{j,t+j-1})+\beta V^h_{t+j}(a_{j,t+j-1},j+1)\} \tag{15}$$

subject to (6).

Individuals solve their perfect foresight dynamic problem by using backward induction. Back to age 1, an individual with disutility index i will choose $s_{i,t}$ based on the criterion as follows:

$$\begin{aligned} s_{i,t}&=c, \quad \text{if } V^c_t(a_{0,t-1}=0,1)-\chi(i)>V^h_t(a_{0,t-1}=0,1),\\ s_{i,t}&=h, \quad \text{if } V^c_t(a_{0,t-1}=0,1)-\chi(i)<V^h_t(a_{0,t-1}=0,1),\\ s_{i,t}&=\text{indifferent} \quad \text{if } V^c_t(a_{0,t-1}=0,1)-\chi(i)=V^h_t(a_{0,t-1}=0,1). \end{aligned} \tag{16}$$

Based on the individual's dynamic program and the schooling choice criterion above, the definition of the competitive equilibrium in this model economy is as follows.

Definition 1. Let $\mathcal{A}=R$, $\mathcal{S}=\{c,h\}$, $\mathcal{J}=\{1,2,\ldots,J\}$, $\mathcal{D}=[0,1]$, and $\mathcal{T}=\{1,2,\ldots,T\}$. Given the age structure $\{\{\eta_{j,t}\}_{j=1}^J\}_{t=1}^T$, a competitive equilibrium is a sequence of individual value functions $V^s_t:\mathcal{A}\times\mathcal{J}\to R$; individual consumption decision rules $C^s_t:\mathcal{A}\times\mathcal{J}\to R_+$; individual saving decision rules $A^s_t:\mathcal{A}\times\mathcal{J}\to\mathcal{A}$ for $s\in\mathcal{S}$ and $t\in\mathcal{T}$; an individual i's period 1 schooling choice $s^*_{i,t}$ for $s\in\mathcal{S}$, $i\in\mathcal{D}$, and $t\in\mathcal{T}$; an allocation of capital and labor (skilled and unskilled) inputs $\{K_t,S_t,U_t\}_{t=1}^T$ for the firm; a price system $\{w^c_t,w^h_t,r_t\}_{t=1}^T$; and a sequence of measures of individual distribution over age and assets $\lambda^s_t:\mathcal{A}\times\mathcal{J}\to R_+$ for $s\in\mathcal{S}$ and $t\in\mathcal{T}$ such that:

1. Given prices $\{w^c_t,w^h_t,r_t\}$, the individual decision rules C^s_t and A^s_t solve the individual dynamic problems (14) and (15).
2. Optimal schooling choice $s^*_{i,t}$ is the solution to the schooling choice criterion in (16) for each individual i.
3. Prices $\{w^c_t,w^h_t,r_t\}$ are the solutions to the firm's profit maximization (9)–(11).
4. The time-variant age-dependent distribution of individuals choosing s follows the law of motion

$$\lambda^s_{t+1}(a',j+1)=\sum_{a:a'\in A^s_t(a,j)}\lambda^s_t(a,j). \tag{17}$$

5. Individual and aggregate behaviors are consistent

$$K_t=\sum_j\sum_a\sum_s\eta_{j,t}\lambda^s_t(a,j)A^s_t(a,j-1), \tag{18}$$

$$S_t=\sum_j\sum_a\eta_{j,t}\lambda^c_t(a,j)\varepsilon^c_j, \tag{19}$$

$$U_t=\sum_j\sum_a\eta_{j,t}\lambda^h_t(a,j)\varepsilon^h_j. \tag{20}$$

6. The goods market clears

$$\sum_{j=1}^J\sum_a\sum_s\eta_{j,t}\lambda^s_t(a,j)C^s_t(a,j)+\sum_{j=1}^4\sum_a\eta_{j,t}\lambda^c_t(a,j)p_{j,t}+X_t=Y_t \tag{21}$$

or

$$C_t+P_t+X_t=Y_t.$$

When ISTC and demographic change both stabilize at some constant level, that is, $q_t=q$ and $n_t=n$, $\forall t$, the economy reaches a steady state. In such a steady state, the age structure, the distribution of individuals over assets and age, and the individual decision rules are all age-dependent but time-invariant. Therefore, we can define the stationary competitive equilibrium accordingly.

4. Parameterization

In this section, we calibrate the model economy to replicate certain properties of the U.S. economy in the pre-1951 initial steady state. More specifically, our strategy is to choose parameter values to match *on average* features of the U.S. economy from 1947 to 1951.[10] It is worth emphasizing that the evolution of the skill premium over the 1951–2000 period is not a target of calibration but the goal for evaluating the model's performance.

[10] We choose the U.S. economy from 1947 to 1951 as the initial steady state based on the observations that both the ISTC and the demographic changes were quite stable during this time period.

4.1. Cohort-specific skill premium

The skill premium data we report in Fig. 1 are the average skill premium across all age groups in a specific year. However, since the model presented here is a cohort-based OLG model, each cohort's college-going decision is based on this cohort's specific lifetime skill premium profile. For example, for the cohort born at time t, the lifetime cohort-specific skill premium is $\{w^c_{t+j-1}/w^h_{t+j-1}\}^J_{j=1}$. In order to understand the mechanism of the schooling decision for each cohort, and more important, as will be shown later, to help pin down the distribution of the disutility cost $\chi(i)$, we need to find the data counterpart of this cohort-specific skill premium.

We use March CPS data from 1962 to 2003, plus 1950 and 1960 census data, to construct the cohort-specific skill premium profiles for the 1948–1991 cohorts. (We choose to end the sample in 1991 because of the quality of the data. The 1991 cohort has only 12-year HSG wage and 8-year CG wage data.) In order to make our results comparable to the literature, we follow Eckstein and Nagypál (2004) in restricting the data (refer to their paper for the details). The sample includes all full-time full-year (FTFY) workers between ages 18 and 65. To be consistent with the model, we look only at high school graduates (HSG) and college graduates (CG). The wage here is the annualized wage and salary earnings. The personal consumption expenditure deflator from NIPA is used to convert all wages to constant 2002 dollars.

Since the CPS is not a panel data set, theoretically speaking, we cannot track specific cohorts from it. However, since it is a repeated cross-sectional data set, we can use a so-called "synthetic cohort construction method" to construct a proxy of a cohort's specific skill premium. Using this method repeatedly for each birth cohort, we have the original data sequences of cohort-specific HSG and CG lifetime wage profiles for the 1948–1991 cohorts. However, due to the time range of the CPS data, some data points are missing for a complete lifetime profile for every cohort. For example, some cohorts are missing at the late-age data points (cohorts after 1962) and some are missing at the early-age data points (e.g., cohorts 1948–1961). We use an econometric method to predict the mean wage at those specific age points and extrapolate the missing data. We predict them by either second- or third-order polynomials, or a conditional Mincer equation as follows:

$$\log[\text{HSGwage(age)}] = \beta^h_0 + \beta^h_1 \text{experience}_h + \beta^h_2 \text{experience}^2_h + \varepsilon^h, \quad \text{experience}_h = \text{age} - 18,$$

$$\log[\text{CGwage(age)}] = \beta^c_0 + \beta^c_1 \text{experience}_c + \beta^c_2 \text{experience}^2_c + \varepsilon^c, \quad \text{experience}_c = \text{age} - 22.$$

The criterion is basically the goodness of fit. We check with the neighborhood cohorts to make sure the predicted value is reasonable. The "rule of thumb" of a hump-shaped profile also applies here to help make choices. As an example, Fig. 6 shows a complete life cycle wage profile of HSGs and CGs for the 1975 cohort by using the prediction from third-order polynomials.

4.2. Distribution of the disutility cost

The distribution of disutility cost $\chi(i)$ becomes very crucial in the computation because it is this distribution that determines the enrollment rate and hence the relative supply of skilled labor in the model. The schooling choice criterion embodied in (16) actually sheds some light on how to compute the distribution of the disutility cost. Note that the person i^* who is indifferent between going to college or not has

$$V^c_t(a_{0,t-1} = 0,1) - \chi(i^*) = V^h_t(a_{0,t-1} = 0,1)$$

that is, her disutility cost is exactly the difference between two conditional value functions. Since the disutility cost is a decreasing function of index i, individuals with disutility index $i > i^*$ go to college. Therefore, for a specific cohort t, if we calculate the difference between two conditional value functions $V^c_t(a_{0,t-1} = 0,1) - V^h_t(a_{0,t-1} = 0,1)$, we obtain the cut-off disutility cost for this cohort. If we also know the enrollment rate of this cohort, it tells us the proportion of people in this cohort who have less disutility than i^* at that specific cut-off point of the disutility cost. In this way, we can pin down one point on the CDF of the disutility cost. Applying this procedure to different cohorts will give us a picture of how the disutility cost is distributed.[11]

Estimating this CDF function involves a fixed-point algorithm, which we describe here step by step. Step 1, we guess the interest rate r under the calibrated discount parameter β and the calibrated preference parameter σ. For each cohort born at time t, we normalize the 18-year-old HSG wage (which is $w^h_t \varepsilon^h_1$ in the model) to one and input the normalized cohort-specific lifetime wage profiles for both HSGs and CGs from the data constructed in Section 4.1. We go through the backward induction of the Bellman equation as described in Section 3.6 to obtain the value function difference $V^c_t(a_{0,t-1} = 0,1) - V^h_t(a_{0,t-1} = 0,1)$ and hence the cut-off disutility cost for every cohort t from 1948 to 1991. By plotting them on the x-axis against enrollment rate data in the same time range (1948–1991) on the y-axis, we have 44 points on the possible CDF of the disutility cost. By assuming that the disutility costs follow a normal distribution, we then estimate the CDF function.[12] Step 2, we feed this estimated CDF in the computation of the stationary equilibrium (see step 4 in Appendix B). We follow the procedure in Appendix B to compute the steady state of the model economy. Step 3, we then

[11] Here we assume the distribution of the disutility cost is stationary.

[12] Heckman et al. (1998) also assume that the "nonpecuniary benefit of attending college" is normally distributed. A more flexible Beta distribution yields a very similar estimated CDF as a normal distribution within the reasonable range of the disutility cost.

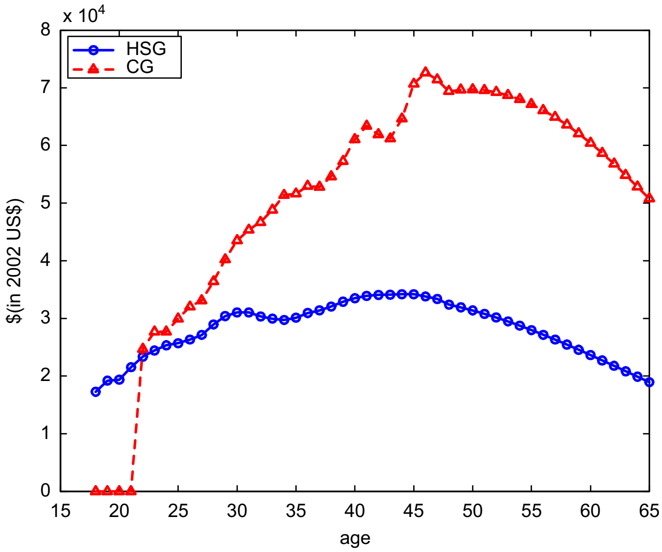

Fig. 6. Life-cycle HSG wage profile: 1975 cohort.

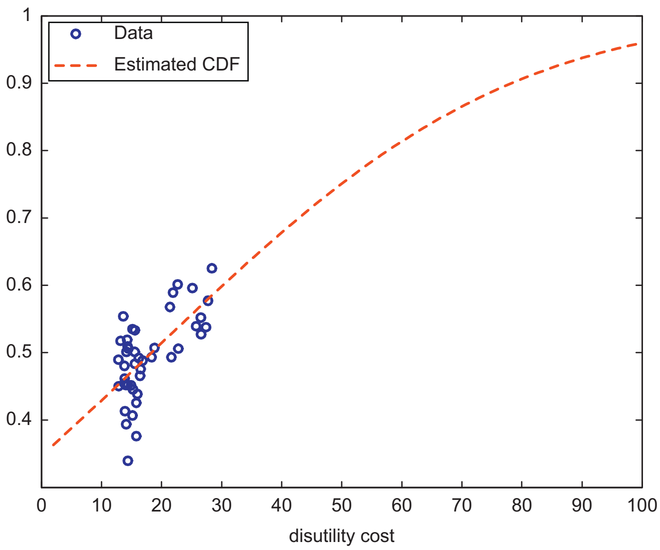

Fig. 7. CDF of disutility cost.

check if the equilibrium interest rate r we obtain in the steady state (based on the estimated CDF given initial guess r) is the same as the guess in step 1. If it is not, we have to change the guess on r and repeat step 1. The iteration stops when the initial guess on r used in calculating CDF in step 1 converges to the equilibrium interest rate in the steady state in step 2. The resulting CDF, shown in Fig. 7, is the estimated CDF of the disutility cost that is used in the benchmark model.

4.3. Demographic

The model period is one year. Agents enter the model at age 18 ($j=1$), work up to age 65 ($J=48$), and die thereafter.

The growth rate of cohort size n that is used in the initial steady state is calculated as the average growth rate of the HSG cohort size from 1948 to 1951, which is 0%.

4.4. Preferences and endowments

We pick CRRA coefficient $\sigma=1.5$, which is in the reasonable range between 1 and 5 and is widely used in the literature (e.g., Gourinchas and Parker, 2002).

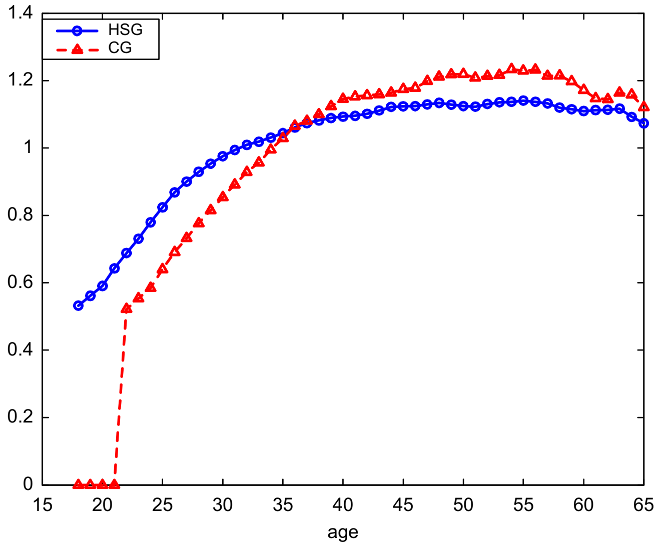

Fig. 8. Age efficiency unit profile.

The age efficiency profiles of high school graduates $\{\varepsilon_j^h\}_{j=1}^J$ and college graduates $\{\varepsilon_j^c\}_{j=1}^J$ are calculated as follows: from the 1962–2003 CPS and the 1950 and 1960 census data we calculate the mean HSG and CG wages across all ages for the time period 1949–2002, then we obtain the mean HSG and CG wages in the same time period for each age group. Thus, the age efficiency profiles are expressed as

$$\varepsilon_j^h = \frac{\text{HSGwage}_j}{\text{HSGwage}}, \quad \varepsilon_j^c = \frac{\text{CGwage}_j}{\text{CGwage}}, \ \forall j = 1, \ldots, 48.$$

The result is shown in Fig. 8. Both profiles exhibit a clear hump shape and reach a peak around age 55. Also notice that $\varepsilon_j^c = 0, \forall j = 1, \ldots, 4$, since we assume CGs never work while in school.

Tuition for the 1951 cohort is the real TFRB charges from 1951 to 1954 as shown in Fig. 5. We divide them by the data of real labor income of age 18 unskilled workers in 1951 and thus convert these four-year tuitions into four ratios. The ratios are then inputted into the model and are multiplied by the model-generated real labor income of age 18 unskilled labor $w\varepsilon_1^h$ to convert back to the model counterpart of the tuitions.

4.5. Production technology

The difference between our production function and the one in KORV (2000) is that we do not distinguish between structures (K_s) and equipment (K_e) as in KORV (2000), so the capital K in our model is just the total capital stock which is the sum of capital equipment and structures. The reason why we use K instead of differentiated K_e and K_s is that in our decentralized OLG setting, the capital is provided through individuals' savings. Individuals cannot distinguish equipment and structures from their savings. We might need a social planner to do so. That significantly increases the complexity of the model and the computation. However, to provide a comparison with the literature two key elasticity parameters in the production function, the coefficient for elasticity of substitution between capital and skilled labor $\rho = -0.495$ and the coefficient for elasticity of substitution between unskilled labor and the capital–skilled labor combination $\theta = 0.401$, are taken directly from KORV (2000). This implies that the elasticity of substitution between capital and skilled labor is 0.67 and the one between unskilled and skilled labor is 1.67. Capital–skill complementarity is satisfied. In Section 8.1, we will do some sensitivity analysis on these two key parameters.

In the initial steady state, both TFP level A and capital productivity B are normalized to unity. ISTC q is also normalized to one. We set the depreciation rate of capital δ to 0.069 by following İmrohoroğlu et al. (1999), who calculate this parameter from annual U.S. data since 1954. Since ISTC stabilizes in the initial steady state, the transformed depreciation rate $\tilde{\delta}$ is equal to δ.

4.6. ISTC

Following GHK (1997) and KORV (2000), in the benchmark model, due to the existence of ISTC, the relative price of capital goods is equal to the inverse of the investment-specific technological change q. Therefore, we can use the relative price of capital to identify ISTC q. We take the NIPA price index of personal nondurable consumption expenditures and services, and the quality-adjusted price index of total investment (equipment and structures) from Cummins and Violante (2002) for the time period 1951–2000. We then divide these two sequences to obtain the data counterpart of q. Finally, we normalize the level of q

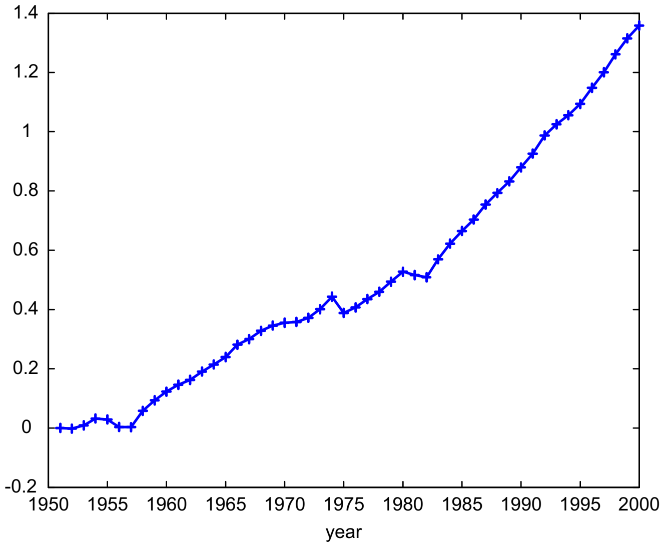

Fig. 9. Investment-specific technological change (log units).

in 1951 to be one. Fig. 9 shows the natural logarithm of the time series of q_t. It was fairly stable before 1957, then started to grow. The average growth rate of q in the 1960s and 1970s was 1.8% and 1.7%, respectively. It has speeded up since the early 1980s. The average growth rate in the 1980s was 3.2% and it was even higher in the 1990s (4.4%).

This leaves four parameter values to be calibrated: the subjective discount rate β, the income share of capital in the capital–skilled labor combination λ, the income share of unskilled labor μ, and the scale factor of the disutility cost sd (see Appendix B for details).[13] We calibrate these four parameters so that the model can replicate, as closely as possible, four moment conditions in the data for the period 1947–1951. These four moment conditions are:

1. Average capital–output ratio 2.67 from 1947 to 1951 (NIPA data).
2. Average income share of labor 72.43% from 1947 to 1951 (NIPA data).
3. Average skill premium 1.4556 in 1949 (census data).
4. Average college enrollment rate 41.54% from 1947 to 1951.

This exercise ends up with $\beta=1.027$, $\lambda=0.645$, $\mu=0.415$, and $sd=2.90$. Table 1 summarizes the parameters used in the model.

The computation method of the steady state is described in detail in Appendix B.

5. Steady-state results

5.1. Initial steady state

In this section, we report the numerical simulations for the stationary equilibrium of the benchmark economy and compare the results with the pre-1951 U.S. data. The macroaggregates that the model generates are shown in Table 2.

The simulations show that the model does well in matching the data. It matches our targets—skill premium (w^c/w^h), enrollment rate (e), capital–output ratio (K/Y), and labor income–output ratio ($(w^cS+w^hU)/Y$) by construction. Additionally, several key macroaggregate ratios, such as the consumption–output ratio (C/Y) and the investment–output ratio (X/Y), are also in line with the U.S. average data. The risk-free real interest rate is 3.33%.

5.2. Comparative static experiments

In this section, we carry out some comparative static exercises to study the effects of the growth rate of cohort size by changing n and the effects of investment-specific technological change by changing B in the steady state. In other words, we compare steady states between different economies with different growth rates of n and B, respectively, while keeping other parameters unchanged as in the benchmark case. We summarize the corresponding results in Tables 3 and 4,

[13] As shown in Section 4.2, since we use the estimated CDF of the disutility cost along with the computed value function difference to determine the enrollment rate in the simulation, there is a scale difference between the CDF that is estimated from the data and the value function difference that is computed based on the parameter values in the model. Scale factor sd is used to take care of this scale difference.

Table 1
Parameter values in the benchmark model.

Parameter	Description	Value and source
J	Maximum life span	48, corresponding to age 65 in real life
$\{\varepsilon_j^s\}_{j=1}^J, s=c,h$	Age efficiency profiles	1962–2003 CPS, and 1950, 1960 census
σ	CRRA coefficient	1.5, Gourinchas and Parker (2002)
θ	Elasticity b/w U and K	0.401, KORV (2000)
ρ	Elasticity b/w S and K	−0.495, KORV (2000)
δ	Depreciation rate	0.069, İmrohoroğlu et al. (1999)
β	Discount rate	1.027
λ	Share of K in production	0.645
μ	Share of U in production	0.415
sd	Scale factor of disutility cost	2.90

Table 2
Macro aggregates in the benchmark economy: initial steady state.

Variable	Model	Data
w^c/w^h	1.4544 (construction)	1.4556 (1949 census data)
e	41.61% (construction)	41.54% (1947–1951 average)
K/Y	2.72 (construction)	2.67 (1947–1951 average)
$(w^hU+w^cS)/Y$	72.17% (construction)	72.43% (1947–1951 average)
C/Y	80.65%	79.57% (1947–1951 average)
X/Y	18.80%	20.17% (1947–1951 average)
r	3.33%	

Table 3
Effect of population growth on steady state.

n (%)	w^c/w^h	e (%)	S/U (%)	BK/S	Rela. supply effect	K–S comple. effect
0 (benchmark)	1.4544	41.61	67.99	5.58		
4.06	1.5105	41.96	61.70	5.30	1.5408	1.4240
−1.57	1.4412	41.52	69.74	5.68	1.4318	1.4631

Table 4
Effect of investment-specific technological change on steady state.

B	w^c/w^h	e (%)	S/U (%)	BK/S	Rela. supply effect	K–S comple. effect
1 (benchmark)	1.4544	41.61	67.99	5.58		
3.73 (2000 level)	1.8805	48.33	89.23	20.47	1.2351	2.2133

respectively. In Table 3, 0% is the average growth rate of the HSG cohort size from 1947 to 1951, which is our benchmark case; 4.06% is the average growth rate of the HSG cohort size from 1952 to 1976, the "baby boom" period; and −1.57% is the average growth rate from 1977 to 1991, the period when n_t continuously decreased. The results show that as the growth rate of the HSG cohort size increases, the skill premium also increases, and vice versa.

Why does the increase in the HSG cohort size cause an increase in the skill premium? The intuition is as follows: an increase in n will change the age structure $\{\mu_j\}_{j=1}^J$ in the economy, skewing it toward younger cohorts. Keeping the enrollment rate unchanged, more individuals from the college-age cohort stay in college. Meanwhile, more people from the college-age cohort also join the labor force as unskilled labor. This results in relatively less *out-of-college* skilled labor in the current labor market, as shown in Table 3.[14] When n increases to around 4%, the relative supply of skilled labor S/U

[14] To formalize this idea, consider a two-period OLG model with young and old agents, in which becoming skilled takes one period. Assuming the enrollment rate e is the constant propensity to go to college among young agents and n is the population growth rate, we then have

$$\left(\frac{S}{U}\right)_t = \left(\frac{S_{old}/U_{old}}{1+U_{young}/U_{old}}\right)_t = \left(\frac{e/(1-e)}{2+n}\right)_t.$$

Clearly higher n leads to a lower S/U ratio.

decreases by 6.3%. This change tends to raise the relative price of skilled labor, which is the skill premium, through the relative quantity effect. However, a change in age structure also has an impact on asset accumulation. People accumulate fewer assets during their early working years. A shift toward younger cohorts in the demographic structure thus decreases the incentive to accumulate assets in the economy. As a result, the capital–output ratio (K/Y) decreases from 2.72 in the benchmark case to 2.52 in the $n=4.06\%$ case. It also leads to a decrease in the effective capital–skilled labor ratio (BK/S). Then, through the capital–skill complementarity effect, it tends to decrease the skill premium. We can further disentangle the relative quantity and capital–skill complementarity effect as in Eq. (12). Suppose that S/U changes from the benchmark value 67.99% to 61.70% in the $n=4.06\%$ case, while we keep BK/S the same as in the benchmark case. Eq. (12) would imply that the skill premium increases from 1.4544 in the benchmark case to 1.5408. On the other hand, suppose S/U remains unchanged, while we change the effective capital–skilled labor ratio BK/S from its benchmark value 5.58 to its value under $n=4.06\%$. Then Eq. (12) would imply that the skill premium decreases from its benchmark value to 1.4240 via only the capital–skill complementarity effect. Quantitatively, the impact of the demographic change on the relative supply of skilled labor dominates that on the relative demand for skilled labor through capital–skill complementarity. Thus, the skill premium increases to 1.5105, which is closer to the one implied by the pure relative supply effect. On the other hand, a decrease in n will make the age structure favor the older cohort and, hence, will increase the relative supply of skilled labor and raise the incentive to accumulate assets. These two impacts again tend to offset each other. Quantitatively, a change in n from 0% to -1.57% slightly decreases the skill premium. Again this is in the same direction as the pure relative supply effect, which shows that a change in n affects the relative supply of skilled labor more significantly.

Since an increase in n raises the skill premium, it also increases the benefits of going to college. The experiment shows that under the $n=4.06\%$ case, the college enrollment rate increases from 41.61% in the benchmark case to 41.96%. On the other hand, a decrease of n from 0% to -1.57% lowers the enrollment rate from 41.61% to 41.52%. Compared to the impact on the skill premium, the effect of demographic change on the college enrollment rate seems very insignificant.

Next, we show the effect of a permanent change in q on the steady state. As shown in Fig. 9, ISTC q_t has increased from 1 in 1951 to 3.89 in 2000. In other words, ISTC has been increasing almost four times over this period. As mentioned in Section 3.5, we can map the sequence of q_t to the changes in the capital productivity level B_t, which translates into a change of capital productivity B_t from its initial steady state value of one in 1951 to 3.73 in 2000. Suppose that the U.S. economy reaches the steady state again after 2000. Keeping other things equal, Table 4 shows the effects of this permanent change from B_{1951} to B_{2000}.

Investment-specific technological change, through capital–skill complementarity, increases the skill premium significantly. The mechanism is as follows: ISTC raises capital productivity B_t and, hence, raises the effective capital stock, B_tK_t. Since capital is complementary to skilled labor, increases in effective capital also raise the demand for skilled labor. As evidence, when B increases from one to 3.73, the effective capital–skilled labor ratio (BK/S) increases from 5.58 to 20.47, almost four times higher. The increase in BK/S alone would raise the skill premium from 1.4544 to 2.2133. However, a rising skill premium gives individuals a stronger incentive to go to college. The enrollment rate increases from 41.61% to 48.33% and hence raises the relative supply of skilled labor from 67.99% to 89.23%. The change in the relative supply of skilled labor alone would reduce the skill premium from 1.4544 to 1.2351. The quantitative results in Table 4 confirm that the first-order impact of ISTC is on the demand side of skilled labor through capital–skill complementarity. This impact dominates the repercussion effect from the relative supply side. Hence, the skill premium increases from 1.4541 to 1.8805 in the model, which is quite close to 1.8357 in the data.

6. Transition path

The comparative static exercises above (especially the one with ISTC) show that we are on the right track in explaining the increases in the skill premium over time. However, to assess the model's performance on matching the time series data of the skill premium in the postwar U.S. economy, comparative static analysis is not enough. One has to solve the model along a time path.

Following the spirit of the computation method in Chen et al. (2006) and Conesa and Krueger (1999), we compute the model along a transition path from the initial pre-1951 steady state toward a final steady state in the far future. The computation algorithm is described in detail in Appendix C.

6.1. Benchmark case

In the benchmark case, we feed into the model the exogenous path of capital-specific technological change $\{B_t\}_{t=1951}^{2000}$ and demographic change embodied in the change in the growth rate of the HSG cohort size $\{n_t\}_{t=1951}^{2000}$. We also feed in the normalized tuition payments $\{p_t\}_{t=1951}^{2000}$.[15] We then assume that capital-specific technological change keeps increasing after 2000 at the average growth rate of the period 1951–2000 until 2020, and it stabilizes at this

[15] As described in Section 4.4 for the steady state, the four-year tuitions faced by each cohort are normalized by dividing them by the age 18 unskilled worker's annual wage of the same cohort in the data. The four ratios then are inputted into the model and are multiplied by the model-generated real labor income of age 18 unskilled labor $w_t\varepsilon_{1,t}^h$ to convert back to the model counterpart of the tuitions.

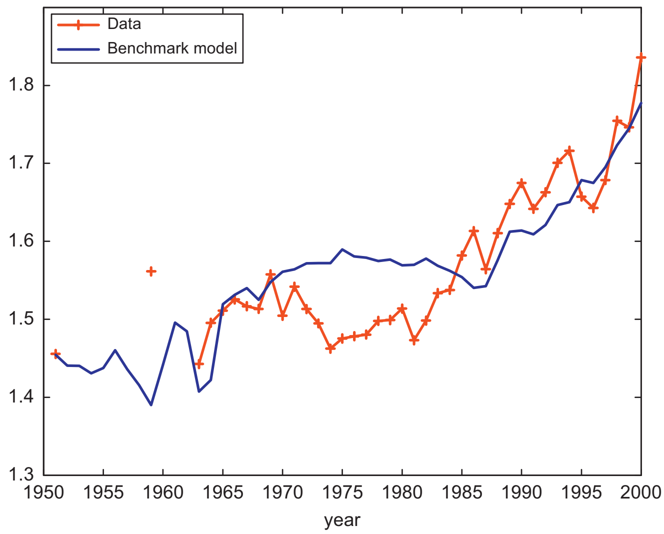

Fig. 10. Skill premium: model vs. data.

constant level until 2050.[16] For simplicity, we also assume that after 2000 there is no demographic change and that the tuition payment is constant at the 2000 level. Since we want to focus on the effect of ISTC, the neutral TFP change has been normalized to unity for all time periods through the transition.[17] We compute the transition path of the benchmark economy between 1951 and 2050 and truncate it to the 1951–2000 period. The results are shown in Fig. 10. Notice that since the skill premium and the college enrollment rate in 1951 have been used for calibration, we can only tell the performance of the model by comparing the model-generated variables with the data after 1951.

In Fig. 10, the simulated skill premium from the benchmark economy captures the increasing trend over the period 1951–2000. From 1951 to 2000, the data show that the skill premium increases from 1.4556 to 1.8357, and the average annual growth rate during these 50 years is 0.49%. In the model the skill premium increases from 1.4544 to 1.7774 and the average annual growth rate is 0.43%. In other words, the model captures about 85% of the increase in the level of the skill premium over the period 1951–2000. From 1963, we have annual data for the skill premium so the comparison between the data and the model's performance is more accurate. From 1963 to 1970, the data show the skill premium increases from 1.4427 to 1.5047 at an average annual growth rate of 0.63%, while the model predicts an increase from 1.4075 to 1.5608 at an average annual growth rate of 0.67%. From 1971 to 1980, the skill premium first decreases sharply but it has started to increase since 1975. The average annual growth rate is almost zero at 0.07%. The model, however, overshoots the data and misses this decline by predicting an almost flat skill premium over this period. The average annual growth rate is 0.06%. The skill premium starts to increase dramatically beginning in 1981 in the data. From 1981 to 2000, the skill premium increases from 1.4730 to 1.8357 at an average annual growth rate of 1.00%, while the model predicts an increase from 1.5696 to 1.7774 at an average annual growth rate of 0.63%. The model captures 56% of the increase in the skill premium during these two decades. Overall, for the three episodes in the "*N*" shape of the skill premium from 1963 to 2000, the model captures the increasing trend of the changes in the skill premium for the period 1963–1970 and 1981–2000. But it fails to replicate the declining part of the skill premium and significantly overshoots the data in the 1970s.

The model also raises the enrollment rate from 41.61% to 49.11% for the period 1951–2000, while the enrollment rate in the data increases from 41.54% to 63.3%, as shown in Fig. 11. In other words, the benchmark model can explain about 35% of the increase in the enrollment rate during this period. Matching the college enrollment rate is not the target of the current model and understanding why the enrollment rate has changed over time is a deeper goal, which is beyond the scope of this paper. The model here is a highly abstract one that excludes many of the important determinants of an individual's schooling choice such as policy changes (e.g., the GI Bill and the Vietnam War) and the shifts in social norms that have especially affected women's decision to go to college (see Goldin, 2006; He, 2011).[18] However, with such a highly stylized model in which the main determinant of college attendance is the net present value of future wages, two exogenous driving forces can still capture more than one-third of the increase in the college enrollment rate from 1951 to 2000.

[16] The results are actually not sensitive to the timing of the final steady state. See Section 8.4 for details.

[17] See Section 8.2 for the sensitivity analysis when the variable neutral TFP change is allowed.

[18] A possible reason why the model does not capture the enrollment rate data well is that the model does not allow college dropout and/or a two-year college program. Individuals who enroll in the model will finish college in four years. The data counterpart of the enrollment rate in the model therefore should be the college completion rate instead of the college enrollment rate. The growth in college degrees conferred has been much slower than the growth in college enrollment (see Turner, 2004), which allows the model to have some potential to better match the enrollment rate data.

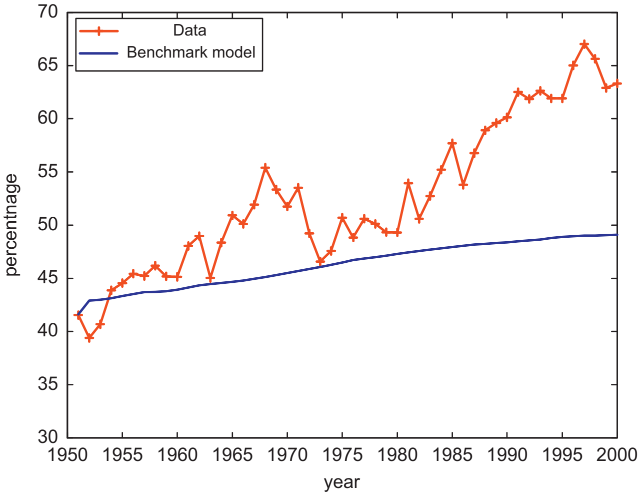

Fig. 11. College enrollment rate: model vs. data.

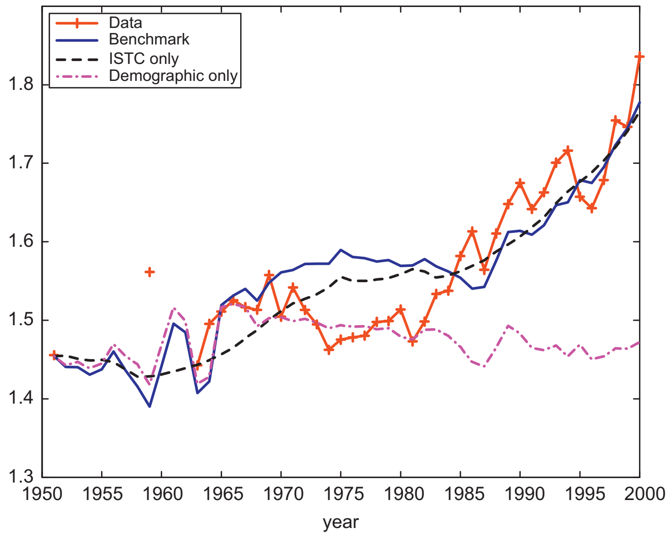

Fig. 12. Skill premium: decomposition.

Of course, a natural question would be: which driving force has a bigger impact on the skill premium and the college enrollment rate? We leave that to the following exercise.

6.2. *Counterfactual decomposition*

To answer the quantitative question raised in the introduction, we conduct the following counterfactual experiments to isolate each exogenous change and investigate its impact on the skill premium and enrollment rate.

We first shut down ISTC by setting $B_t = 1$ for all the years during the transition path, so the only exogenous force remaining is the demographic change. The results are shown in Fig. 12 with the legend "Demographic only." Fig. 12 shows that the model fits the skill premium data fairly well from 1963 until 1980. And more important, it generates the declining skill premium during the 1970s. However, it cannot capture the dramatic increase since 1980 as shown in the data. More specifically, from 1963 to 1970, the model generates a 0.86% average annual growth rate in the skill premium, which can explain 138% of the average annual growth rate in the data for this period. From 1971 to 1980, the data show that the skill premium first decreases and then increases with an average annual growth rate close to zero over the whole decade, while the model generates an average annual growth rate of -0.17%. More important, the skill premium in the model decreases from 1.4994 in 1971 to 1.4791 in 1980, which fits the data better than the benchmark case. However, from 1981, the model predicts a slight decrease in the skill premium (-0.02% per year), in contrast to the dramatic increase shown in the data.

When ISTC is shut down, the model generates little variation in the enrollment rate. As shown in Fig. 13 with the legend "Demographic only," from 1951 to 2000, the model predicts that the enrollment rate slightly decreases from 41.61% to 40.99%, while the data reflect an increase from 41.54% to 63.33%. The average annual growth rate of the enrollment rate is 0.95% in the data, while in the model it is −0.03%. Hence, demographic change does not have a significant effect on the enrollment rate over this period. This confirms the results obtained from comparing steady states in Section 5.2.

Next, we shut down the demographic change by setting $n_t = 0$ for all the years during the transition path. What remains is only ISTC. The results are shown in Fig. 12 with the legend "ISTC only." Overall, the "ISTC only" case shows that the skill premium keeps increasing from 1.4544 in 1951 to 1.7659 in 2000. It captures about 82% of the increase in the skill premium for the whole period. For the period 1963–2000 we have annual data, "ISTC only" increases the skill premium at an average annual growth rate of 0.55%, which captures about 81% of the average annual growth rate in the data over the same period. Compared to the average annual growth rate of 0.11% in the "Demographic only" case, it is obvious that ISTC is much more important in driving the skill premium over the period 1963–2000. Focusing on the three episodes, from 1963 to 1970, the average annual growth rate of the skill premium in the model is 0.66%, which is smaller than the average annual growth rate of 0.86% in the "Demographic only" case. From 1971 to 1980, the data show that the skill premium first decreases and then increases with an average growth rate close to zero, while the model goes in the other direction to predict a growth rate of 0.31% on average. However, after 1981, the "ISTC only" model generates a 0.63% average annual growth rate in the skill premium over the period 1981–2000, which can explain about 63% of the increase in the skill premium in the data. As shown in Table 5, compared to the "Demographic only" case, the increase in the skill premium is entirely driven by the ISTC after 1980.

Fig. 13 also shows the impact of ISTC alone on the college enrollment rate. In the model, the enrollment rate increases from 41.61% in 1951 to 49.13% in 2000, almost the same as in the benchmark model. The increase in the enrollment rate over the period, therefore, is entirely driven by the ISTC.

Since the effective capital–skilled labor ratio (BK/S) and the relative supply of skilled labor (S/U) are the two major determinants of the skill premium in the model (see Eqs. 12 and 13), we also present simulations of both ratios in panel A and B of Fig. 14 respectively. Panel A in Fig. 14 shows the BK/S ratio in the three model cases and the data counterpart of this ratio from KORV (2000). The data for aggregate capital is the sum of the quality-adjusted real stock of equipment and structures from KORV (2000). We normalize the ratio to be equal to the ratio in the benchmark model for 1963 for an easier comparison. Notice that KORV data end in 1992. Overall, the benchmark model fits the data fairly well except that

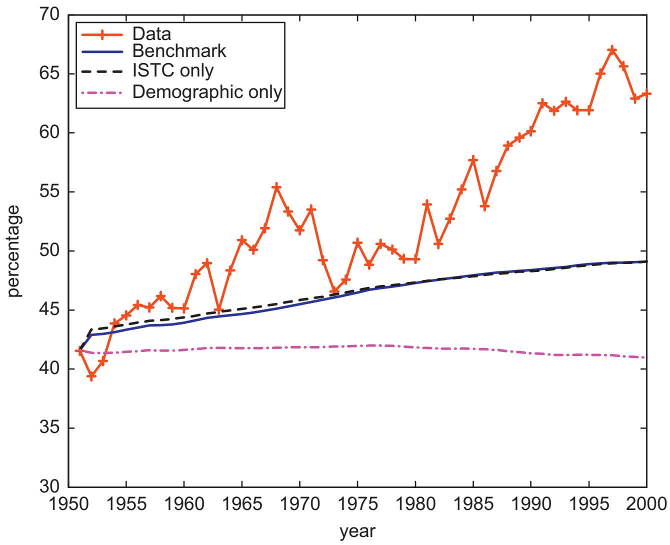

Fig. 13. College enrollment rate: decomposition.

Table 5
Average annual growth rate of the skill premium: model vs. data.

Period	Data (%)	Benchmark (%)	Demographic (%)	ISTC (%)
1963–2000	0.68	0.64	0.11	0.55
1963–1970	0.63	1.51	0.86	0.66
1971–1980	0.07	0.06	−0.17	0.31
1981–2000	1.00	0.63	−0.02	0.63

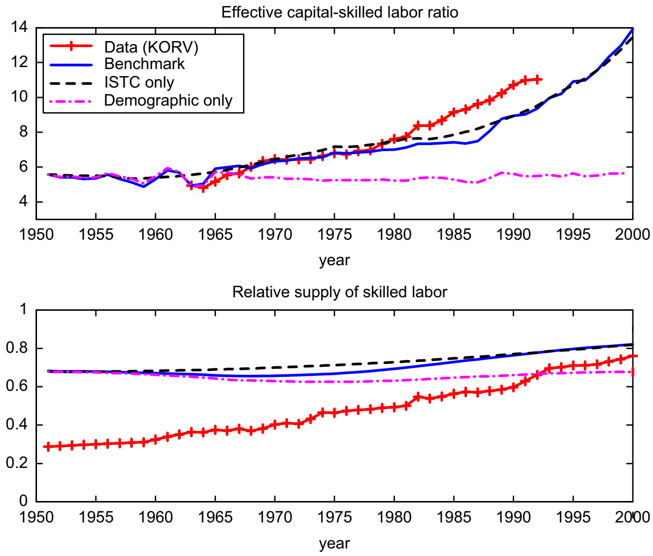

Fig. 14. BK/S and S/U ratio: decomposition.

the model underestimates the data since 1981. And the ratio in the "ISTC only" case closely follows the one in the benchmark model. As discussed in Section 5.2, the baby boom might reduce the skill premium via less capital accumulation and thus less demand for skilled labor. This channel might potentially help to explain the decline of the skill premium in the 1970s. However, the "Demographic only" case shows that the BK/S ratio only slightly decreases in the 1970s. This channel is not quantitatively strong enough to counter the impact from ISTC to drag the skill premium down.

Panel B in Fig. 14 shows the relative supply of skilled labor in the benchmark and the two decomposition cases and the data counterpart. The data show that the relative supply of skilled labor has increased from 28.69% in 1951 to 75.95% in 2000, around 2.6 times. The benchmark model predicts an increase from 67.99% to 81.93%.[19] In other words, the benchmark model only predicts about 30% of the increase in the relative supply of skilled labor due to its difficulty in replicating the college enrollment rate. The "ISTC only" model shows a continuously rising S/U ratio by generating an increase in the enrollment rate over time, which is consistent with the trend in the data. In contrast, the "Demographic only" case predicts a decreasing S/U from 1951 to 1976 and an increasing S/U since then. This is consistent with the mechanism mentioned in Section 5.2: the "baby boom" decreases S/U, while the "baby bust" does the opposite. Although this channel might potentially contribute to the increase in the skill premium from the early 1960s to 1970, as the figure shows, the data of S/U ratio, however, show a steadily increasing trend during that period. This again confirms that the quantitative importance of the demographic change for the evolution of the skill premium is limited.

7. Experiments with exogenous labor supply

As mentioned in the introduction, the benchmark model extends KORV (2000) into a general equilibrium setup by adding two more layers on top of their framework. The first layer embeds ISTC in a general equilibrium framework and hence endogenizes the capital accumulation in KORV's production technology. The second one endogenizes the college entry decision and hence endogenizes the relative supply of skilled labor. In this section, we want to address two questions related to the second layer, i.e., endogenous labor supply in the benchmark model.

7.1. Counterfactual experiment with constant enrollment rate

In order to quantify the role of endogenous college choice for the dynamics of the skill premium, we conduct a counterfactual experiment to shut down the schooling choice. We assume the college enrollment rate stays constant in the level of data in the initial steady state, which is 41.54%. We then recalibrate the initial steady state and rerun the transition path exercise as in Section 6.1. Fig. 15 reports the results.

In panel A of Fig. 15, "Benchmark" replicates the skill premium generated in the benchmark model as in Section 6.1. "Fixed enroll" shows the skill premium in the model with the constant enrollment rate at 41.54%. Without the effect from

[19] The reason why the benchmark model predicts the relative supply of skilled labor of 67.99%, which is much higher than that number in the data, is because in the initial steady state we have to match the college enrollment rate of 41.54% in the data. And since it is a steady state, we have the same enrollment rate for each cohort. In the data, for generations before 1951, they had much lower college enrollment rates than that in 1951, which translate into a lower relative supply.

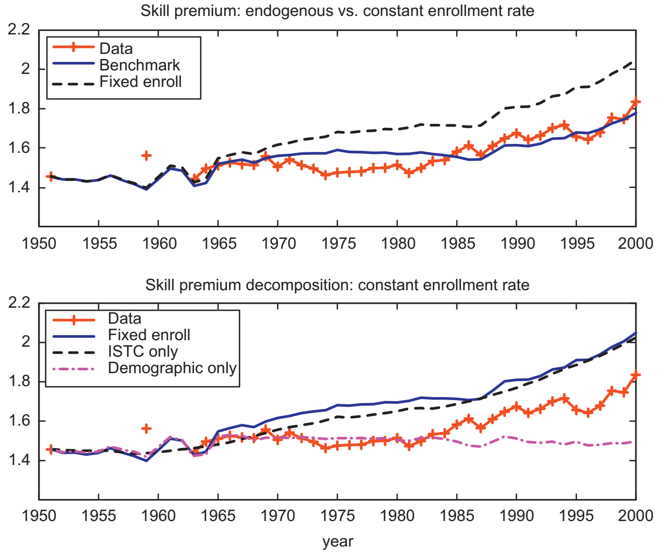

Fig. 15. Skill premium: constant enrollment rate.

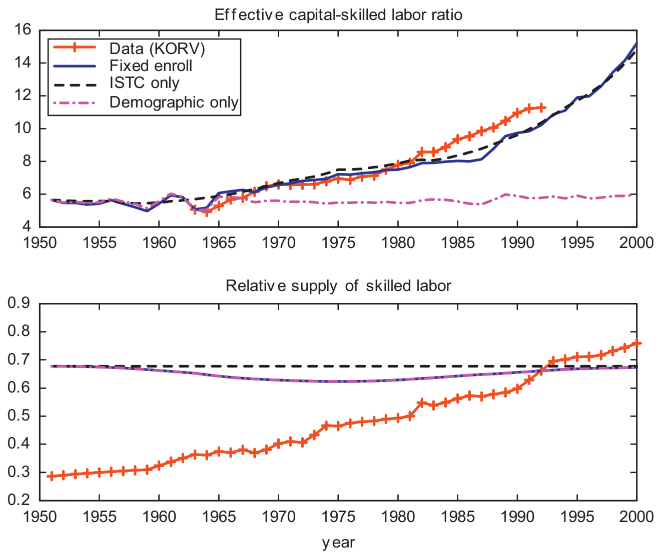

Fig. 16. *BK/S* and *S/U* ratio: constant enrollment rate.

the increasing relative supply generated by the endogenous college choice, the "Fixed enroll" case predicts that the skill premium increases from 1.46 in 1951 to 2.05 in 2000, a performance much worse than the benchmark model. The endogenous college choice is crucial in replicating well the dynamics of the skill premium in the postwar U.S. economy.

Notice that we still feed both ISTC and the demographic change in the model with constant enrollment rate. In panel B of Fig. 15, we rerun the decomposition exercise in Section 6.2 to the "Fixed enroll" model. The exercise replicates the main results in Section 6.1 for the benchmark model. ISTC is the main driving force behind the increasing skill premium, while the demographic change plays a much more limited role quantitatively in the evolution of the skill premium.

To further understand the underlying mechanism to determine the skill premium, we report the effective capital–skilled labor ratio (BK/S) and the relative supply of skilled labor (S/U) in different cases in Fig. 16. Panel A shows BK/S ratio. As in the benchmark model (see Fig. 14), "ISTC only" closely tracks the ratio in the "Fixed enroll" model for most of the time period 1951–2000. And both capture the data fairly well, while the "Demographic only" case shows little variation in the BK/S ratio over time and completely goes against the trend of data after 1967. Panel B in Fig. 16 reports the S/U ratio. Since we shut down the college choice in the model, the change in the relative supply of skilled labor only comes from the change in age shares caused by the demographic change. The S/U ratio in both the "Fixed enroll" and the "Demographic only" cases coincides. And exactly as the intuition developed in Section 5.2 predicts, the "baby boom" reduces the relative supply of skilled labor by reducing the out-of-college skilled labor. The S/U ratio decreases from 1951 to 1976 when n_t increases over time. It increases since then when the "baby bust" hits the economy and n_t decreases continuously. Finally, without the effect of demographic change and facing a constant enrollment rate, "ISTC only" generates a constant S/U ratio. This confirms that the impact of ISTC on the relative supply of skilled labor comes solely from its effect on college choice.

7.2. Experiment with inputted labor supply

From Fig. 14, it is clear that the benchmark model has done a better job in the first layer (embedding ISTC in a general equilibrium framework) than the second one (endogenizing college choice and hence the relative supply of skilled labor). The experiment in Section 7.1 shuts down the endogenous college choice. However, the demographic change still exists there, and it affects the relative supply of skilled labor, as the evidence in panel B of Fig. 16 shows. In this section, we conduct a counterfactual experiment to further decompose these two layers by shutting down the impact of the demographic change on the relative supply of skilled labor as well. In this experiment, instead of endogenizing the school choice as in the benchmark model, we input exogenously the relative supply of skilled labor from the data as in Fig. 1 into the model. In other words, we shut down the second layer in the model completely and focus only on the first one. We then recalibrate the model and compute the transition path. The simulated skill premium is reported in panel A of Fig. 17 with the legend "Fixed supply."

Replicating the relative supply of skilled labor by construction, the model predicts that the skill premium actually decreases from 1.4569 in 1951 to 1.1363 in 2000. It is not surprising because the original benchmark model with endogenous college attendance choice significantly underestimates the relative supply of skilled labor and hence the "relative quantity effect," which has a negative impact on the skill premium as shown in Eq. (13). We also redo the decomposition exercise as in Section 6.2. Since we have a fixed labor supply here, the "ISTC only" case is able to isolate the impact of changing age structure (via variable n_t) on capital accumulation as described qualitatively in Section 5.2, because the only difference between the "Fixed supply" and the "ISTC only" case is that we have a variable growth rate of cohort size n_t (and hence changing age shares) in the former case. As shown in panel A of Fig. 17, for most years before 1988, the "ISTC only" case generates a slightly higher skill premium than the benchmark case. This confirms the mechanism mentioned before: when baby boomers enter the labor market, the age structure shifts toward younger cohorts. The young individuals, however, accumulate less capital in the earlier stage of their life cycle. An increase in n_t hence decreases aggregate capital in the economy because of this shift in age structure. It in turn decreases the skill premium via the "capital–skill complementarity" effect. Our exercise here shows that the mechanism, although it is in the right direction, is not quantitatively important. What is quantitatively important is that when we shut down the ISTC in the model, as shown in the figure, the "Demographic only" case generates a much worse skill premium compared to the data. The difference between the "Benchmark" and "Demographic only" case isolates the quantitative impact of ISTC alone on the skill premium when the labor supply is fixed. The finding in the endogenous labor supply scenario is confirmed here: ISTC is the key element driving the skill premium and its impact speeds up over time.

Since the relative supply is fixed in the model, the only endogenous term that determines the change in skill premium in Eq. (13) is the effective capital–skilled labor ratio BK/S. Panel B of Fig. 17 compares this ratio in three cases with the data from KORV (2000). Similar to panel A of Fig. 14, the model is still able to capture a significant part of the increase in BK/S

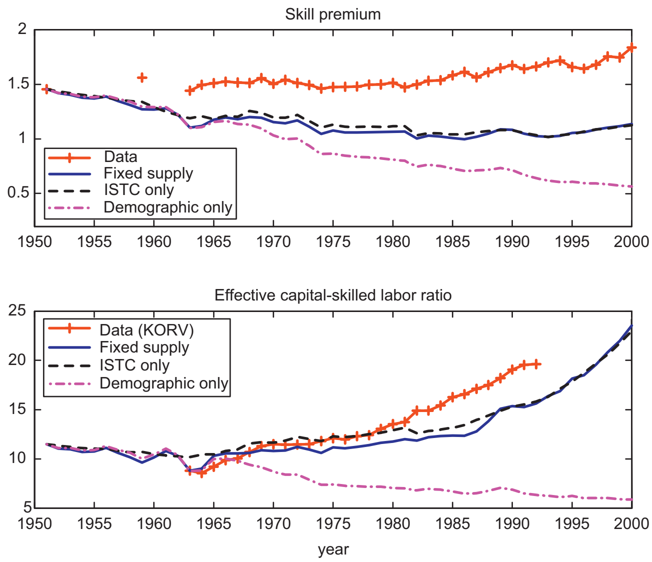

Fig. 17. Model with exogenously inputted labor supply.

data over time due to the existence of ISTC. If we shut down the ISTC, which is the engine of the capital accumulation, with the exogenously increasing relative supply of skilled labor, BK/S decreases significantly as indicated in the "Demographic only" case. Notice that with an exogenously inputted labor supply, the difference in the BK/S ratio between the "Fixed supply" and the "ISTC only" case is able to isolate the quantitative importance of the second channel that the baby boom affects the skill premium, i.e., the change in capital accumulation is solely due to the change in age shares, which in turn is caused by the demographic change. Our theory predicts that the baby boom will reduce capital accumulation since young people save less. That is indeed the case here in panel B, since without variable n_t, the BK/S ratio is higher for most of the time during the baby boom period in the "ISTC only" case.

Comparing Figs. 12 and 17, one can see that the relatively better performance of the original benchmark model in terms of matching the skill premium in the data comes from its underestimation of the relative quantity effect. When the relative supply of skilled labor is inputted exogenously from the data, the capital–skill complementarity effect is not strong enough to drive up the skill premium as in the data. A possible reason could lie in the two key elasticity parameters θ and ρ. KORV (2000) estimate these two parameters in a partial equilibrium framework when four inputs in the production function, S, U, equipment K_e, and structures K_s, are all exogenously inputted from the data. Compared to KORV (2000), the current paper adopts a general equilibrium framework. Therefore, all inputs are endogenous. We also have total capital stock K instead of differentiated K_e and K_s in the current model, which makes the elasticity parameters harder to map to the ones in KORV (2000). The values of θ and ρ affect both physical and human capital accumulation. As shown in Eq. (13), the values of parameter θ and ρ obviously also affect the magnitude of change in the skill premium. When θ is higher, keeping other things equal, it magnifies the capital–skill complementarity effect and shrinks the relative quantity effect simultaneously. How to estimate these two key parameters in a general equilibrium framework is beyond the scope of the current paper. However, in an unreported exercise, if we keep $\rho=-0.495$, but pick $\theta=0.79$, which is the value used in He and Liu (2008), in the current benchmark model with exogenously inputted labor supply, the model predicts that the skill premium increases from 1.4561 in 1951 to 2.1782 in 2000. It actually overshoots the data. With $\theta=0.79$ and $\rho=-0.495$, in Eq. (13), the weight before the term for the "capital–skill complementarity" effect $\theta-\rho$ is much higher than its value under the benchmark parameter values, and the weight before the term for the "relative quantity" effect $\theta-1$ is lower (recall that θ has to be less than 1). Therefore, it is not surprising that it generates a much higher and rising skill premium compared to the benchmark case in Fig. 17.

Another possible reason why the benchmark model with exogenously inputted labor supply significantly underpredicts the evolution of the skill premium is that the parameter values of θ and ρ that KORV (2000) estimate are based on the quality-adjusted capital equipment, not the total capital stock as used in the current paper. ISTC on equipment only is much faster than ISTC on total capital stock. For example, data from Cummins and Violante (2002) show that ISTC on equipment has increased from 1 in 1951 to 8.10 in 2000, more than double than ISTC on total capital stock. If we input the data sequence of ISTC on equipment only into the model, even with the same parameter values of θ and ρ as used in the current paper, one would expect the "capital–skill complementarity" effect to be much stronger than in Fig. 17, which will bring the model simulation much closer to the data. Of course, how to distinguish capital equipment and capital structure in the OLG setting is a challenging question.

8. Sensitivity analysis

In this section, we show that the paper's results are robust to the alternative values of key parameters, including neutral technological change, increase in life expectancy, and the timing of the final steady state. For each experiment, we recalibrate four parameters: β, λ, μ, and sd, while keeping other parameters unchanged as in Table 1. Therefore, the difference in the skill premium generated by each experiment and the benchmark model quantifies the impact of the change embodied in the experiment.

8.1. Elasticity of substitution

Since the two elasticity parameters in the production function, θ and ρ, are the key parameters in the model, we verify the sensitivity of the analysis to different values of these two parameters by running two experiments.

The current paper uses the values of θ and ρ from KORV (2000). However, as pointed out before, in KORV (2000) it is equipment that is complementary to the skilled labor, while in this paper implicitly both structure and equipment are complementary to skilled labor, since we only have total capital stock in the production function. Therefore, simply adopting KORV's parameter for ρ, which is a key parameter determining the elasticity between equipment and skilled labor in their paper but capital and skilled labor in the current paper, might exaggerate the role of the ISTC through the capital–skill complementarity channel, since we know that the structure should be at least much less complementary to skilled labor. So the first experiment we run here is to test model results to different values of ρ, which leads to less complementarity between capital and skilled labor.

Given the same value of $\theta=0.401$, we now choose three different values of ρ: -0.40, -0.30, and -0.20, respectively. capital–skill complementarity $\theta-\rho$ thus decreases from its benchmark value 0.90 to 0.80, 0.70 and 0.60. The skill premia generated under different values of ρ are reported in Fig. 18.

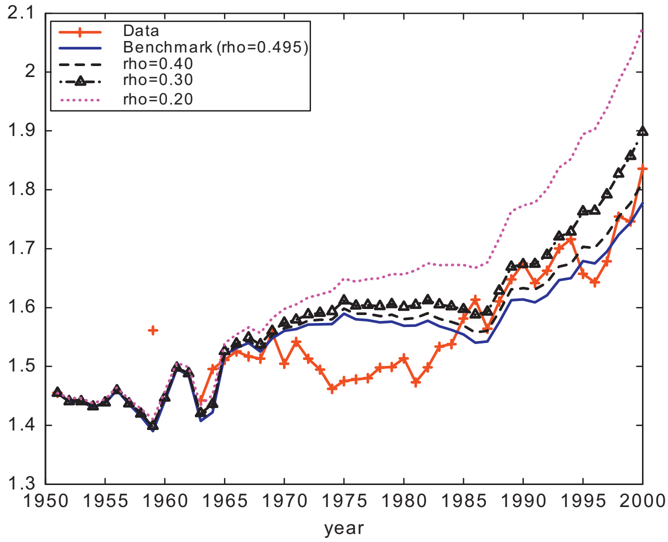

Fig. 18. Skill premium: different ρ.

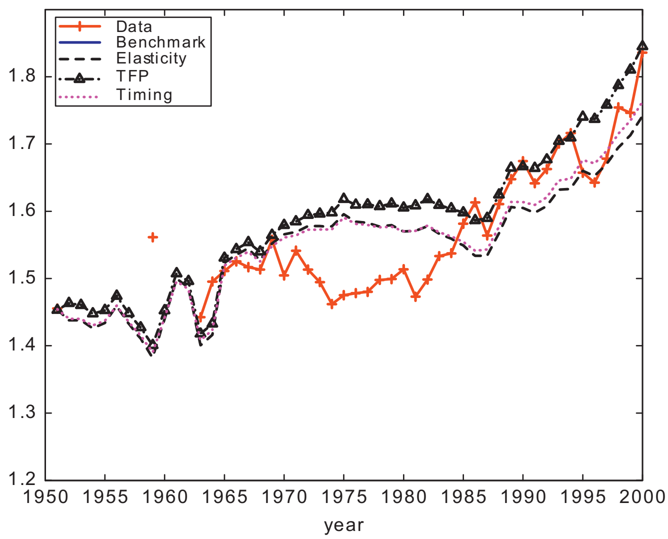

Fig. 19. Skill premium: sensitivity analysis.

Generally speaking, different values of ρ generate a similar increasing trend of skill premium over time, with the exception of $\rho = -0.20$, which generates a significantly higher skill premium than other cases. And, surprisingly, higher ρ, which means less capital–skill complementarity, leads to a higher skill premium. The reason lies in Eq. (13), which determines the evolution of the skill premium in the model. With higher ρ, although capital–skill complementarity $\theta - \rho$ is lower, but higher ρ also makes the term $(BK/S)^{\rho}$ larger, which cancels the decreasing capital–skill complementarity $\theta - \rho$.

In the second experiment, we use the values $\theta = 0.33$ and $\rho = -0.67$, which are taken from Fernandez-Villaverde (2001), since we share the same specification of the production function. This implies that the elasticity of substitution between capital and skilled labor is 0.60, and the one between unskilled and skilled labor is 1.49. We then recalibrate the model according to these new values. All of the comparative static and transition path results are similar to the benchmark results shown here. For example, the skill premium increases from 1.4543 to 1.7417 for the 1951–2000 period as shown in Fig. 19 with the legend "Elasticity."

8.2. Neutral TFP change

So far the model keeps the total factor productivity at a constant level and hence excludes the neutral technological change. Since the TFP change does not enter into Eq. (13), which determines the dynamics of the skill premium, we should not expect that including TFP growth would change the results significantly. As a robustness check, we carry on an experiment that allows TFP to grow at a rate of 0.15% along the transition path. Including the neutral TFP change slightly increases the skill premium. From 1951 to 2000, the skill premium increases from 1.4534 to 1.8449 as shown in Fig. 19 with the legend "TFP," while in the benchmark model, it increases from 1.4543 to 1.7774.

8.3. Increasing life expectancy

Demographic changes were not only happening in terms of fluctuations in the fertility rate. In 1950 life expectancy at birth in the U.S. was only 68.2 years. It was 77 years in 2000. To take into account the effects of this demographic dimension, we do the following experiment. In an extended model with retirement and Social Security (people retire at age 66 and live up to age 100), we first use the survival probability taken from the U.S. Life Table for the 1949–1951 period to calibrate the model. It predicts that the skill premium in the initial steady state is 1.4562. We then keep all of the other parameters unchanged except that now the survival probability is replaced by the data taken from the U.S. Life Table for the 1999–2001 period. Although the survival probability has increased substantially over these 50 years, which leads to the increase in life expectancy, the model predicts that the skill premium changes only slightly, to 1.4218. Given that the survival probability for the period 1999–2001 is the upper bound of the changing survival probabilities for the transition period, this experiment implies that increasing life expectancy would not change our results significantly.[20]

8.4. Timing of the final steady state

Finally, we test the robustness of the model to the timing of the final steady state. In the current model, we set it in an arbitrary way so that the economy reaches the final steady state after 2050. However, the choice of this timing does not significantly affect the results. For example, if the model reaches the final steady state right after 2000, e.g., 2005, we still obtain results almost identical to those in the benchmark model. The skill premium increases from 1.4543 to 1.7632 for the 1951–2000 period as shown in Fig. 19 with the legend "Timing."

9. Conclusion

The skill premium (college wage premium) in the U.S. increased in the 1950s and 1960s, decreased in the 1970s, and has increased dramatically since 1980. What are the driving forces behind this "*N* " shape? The previous literature proposes several explanations, including skill-biased technological change (SBTC) and demographic change. However, less attention has been paid to investigating the relative importance of each driving force to the evolution of the skill premium. In this paper, we establish and compute a general equilibrium overlapping generations model with endogenous schooling choice to answer an important quantitative question: what percentage of the change in the skill premium for the postwar period in the U.S. can be explained by demographic change and investment-specific technological change (ISTC), respectively?

In this model, ISTC and demographic change drive the equilibrium outcomes of the skill premium by dynamically affecting the relative demand and supply of skilled labor. ISTC, through the key feature of capital–skill complementarity in the production technology, increases the relative demand of skilled labor and thus raises the skill premium. In turn, the rising skill premium encourages skill formation and increases the relative supply of skilled labor. In contrast, demographic change affects the age structure in the economy. A change in the age structure has a direct impact on the relative supply of skilled labor. In addition, since people have different saving tendencies along the life-cycle, a change in the age structure also influences the relative demand for skilled labor through changing asset accumulation in the economy. The ultimate effects of these two forces on the skill premium (and college enrollment rate) depend on the quantitative magnitude of both demand and supply effects.

We calibrate the model to match the U.S. data for the period 1947–1951 as the initial steady state. Then, by feeding in the ISTC data from Cummins and Violante (2002) and the growth rate of the HSG cohort size from 1951 to 2000, we conduct perfect foresight deterministic simulations to compare with the data of the 1951–2000 period and counterfactual decomposition experiments to identify the effects of each force.

We find that ISTC plays a dominant role in driving the dramatic increase in the skill premium. It alone captures about 82% of the increase in the skill premium for the period 1951–2000, while the quantitative importance of the demographic change to the evolution of the skill premium is limited, especially after 1980. In addition, we find that ISTC can explain about 35% of the increase in the college enrollment rate for the period 1951–2000, while demographic change does not have a significant effect on the college enrollment rate over time.

Due to the simple structure in modeling college entry and the attrition decision, the current paper has done a less successful job in replicating the entire increase in the college enrollment rate as observed in the data. Understand what missing feature is crucial in capturing the enrollment rate data poses an interesting question for future research.

[20] Restuccia and Dandenbroucke (2010) find that the effect of changing life expectancy on educational attainment is not quantitatively important.

Acknowledgments

This paper is based on the first chapter of my Ph.D. dissertation at the University of Minnesota. I am particularly grateful to my advisor Michele Boldrin for his invaluable advice and encouragement, and to V.V. Chari for his continuously insightful comments. I am also indebted to my dissertation committee members Zvi Eckstein and Selo İmrohoroğlu for their help during the various stages of this work. I thank Kaiji Chen, James Feigenbaum, Chris Flinn, Lei Guo, Ayse İmrohoroğlu, Larry Jones, Ken Judd, Dirk Krueger, Per Krusell, Zheng Liu, Fabrizio Perri, Alice Schoonbroodt, an anonymous referee, and seminar participants at the BCJ Macro-Workshop at the University of Minnesota, the 2005 Chicago-Argonne Institute on Computational Economics, the 2006 North American Summer Meetings of the Econometric Society, the Sixth Villa Mondragone Workshop in Economic Theory and Econometrics, the University of Hawaii, the Industrial Relations Center at the Carlson School of Management at the University of Minnesota, the University of Notre Dame, Wayne State University, Concordia University, the 2007 Midwest Macroeconomics Meetings, and the 2007 SED Meeting for their helpful comments. I also thank D. Autor, D. Acemoglu and Z. Hercowitz for providing me with relevant data and Sally Burke for editorial assistance. All remaining errors are my own.

Appendix A. Data

The skill premium used in Fig. 1 is the ratio of the real mean annualized wage of CGs and HSGs from the CPS for 1962–2003 and the 1950 and 1960 census. Following the literature such as Katz and Autor (1999) and Acemoglu (2003), the data counterpart of the relative supply of skilled labor (S/U) is the ratio of annual hours worked of CG equivalents and HSG equivalents, where college equivalents$=$CG$+0.5\times$ workers with some college, and HSG equivalents$=$HSG$+0.5\times$ workers with some college. The data are taken from the CPS for 1962–2003 and the 1950 and 1960 census. See Section 4.1 for further details on sample selection.

Data on HSG cohort size in Fig. 2 are from the National Center for Education Statistics (NCES): *Digest of Education Statistics* (*DES*) 2002, Table 103. Data for 1941, 1943, and 1945 come from *DES* 1970, Table 66. In this figure "year" refers to school year; for example, 1939 refers to the school year 1939–1940.

The population data for 18-21 years old in Fig. 3 come from different sources. Data for 1970–2000 are from NCES: *DES* 2002, Table 15. Data for 1960–1969 are from NCES: *DES* 1995. Data for 1955–1959 are from the Standard Education Almanac 1968, Table 1.

College enrollment rates of HSG for 1960–2001 in Fig. 4 are from NCES: *DES* 2002, Table 183. Data for 1948–1959 are calculated by the author. To construct them, first we take the 1948–1965 data of first-time freshmen enrolled in institutions of higher education (from NCES: *DES* 1967, Table 86), divided by the HSG cohort size as in Fig. 2. Since first-time freshmen are not necessarily recent HSG, we use the overlapped years 1960–1965 to calculate the average difference between our calculation and the true data, then adjust our calculation for the 1948–1959 period according to this difference.

Data on average TFRB charges in Fig. 5 are constructed as follows. First, we obtain data for estimated average charges to full-time resident degree-credit undergraduate students between 1956–1957 and 1966–1967 from the Standard Education Almanac 1969, Table 120; 1967–1968 to 1973–1974 from the Standard Education Almanac 1981–1982, pp. 231–232; 1974–1975 to 1983–1984 data from the Standard Education Almanac 1984–1985, pp. 328–329; 1984–1985 to 2003–2004 data from "The Trends in College Pricing 2003," the College Board, Tables 5a, 5b. Data for 1948–1955 are from the Standard Education Almanac 1968, Table 102: "Estimated Costs of Attending College, Per Student: 1931–1981". To make it consistent with the data after 1955, we use the overlapped 1956 data to adjust. Second, we focus only on public or private four-year institutions. We obtain the TFRB charges for those institutions. Third, we calculate the enrollment share of public and private four-year institutions. For the 1948–1964 data, we obtain the total fall enrollment in degree-granting institutions by control of institution (private vs. public) from NCES: *DES* 2002 Table 172, noting that it applies to all higher education institutions. Then, from Table 173, we have total fall enrollment in degree-granting institutions by control and type of institution from 1965 to 2000. Fourth, we weight the average TFRB charges of public and private four-year institutions by enrollment share, then use the personal consumption expenditure deflator from NIPA to convert them into constant 2002 dollars. Finally, the third-order moving average method is used to smooth the data.

The construction of the data for the cohort-specific skill premium is in the text. (See Section 4.1.)

Appendix B. Algorithm to compute the stationary equilibrium

Given the parameter values as shown in Table 1, we compute the stationary equilibrium as follows:

1. Guess the initial values for capital stock K_0 and initial enrollment rate e_0.
2. Given the initial guesses, calculate the skilled labor S_0 and the unskilled labor U_0. Notice that for every j, $\sum_a \lambda^c(a,j) = e_0$ and $\sum_a \lambda^h(a,j) = 1-e_0$, so by Eqs. (19) and (20) we have

$$S_0 = e_0 \cdot \sum_j \eta_j \varepsilon_j^c, \quad U_0 = (1-e_0)\cdot \sum_j \eta_j \varepsilon_j^h.$$

Given all the inputs, from the firm's FOCs (9)–(11), we can compute the interest rate r and wage rates w^c and w^h.

3. Discretize the asset level $-b \leq a \leq a_{\max}$ (make sure that the borrowing limit b and the maximum asset $a_{\max}$ will never be reached). Given prices $\{w^c, w^h, r\}$, feed in the normalized tuition data. By using backward induction (remember that $a_J = 0$), we can solve the conditional value function $V^s(a,1)$ for $s = c,h$, and therefore obtain the cut-off disutility cost $\chi(i^*) = (V^c(a=0,1) - V^h(a=0,1))/sd$, where sd is the scale factor of the disutility cost, which is calibrated to replicate the enrollment rate data.
4. From the estimated CDF function of the disutility cost in Fig. 7, corresponding to $\chi(i^*)$, we obtain the new enrollment rate e_1. Check the convergence criterion ($|e_0 - e_1|/e_0 \leq tol_e$). If it is not satisfied, update it by the relaxation method

   $$e_2 = \kappa_e e_0 + (1-\kappa_e)e_1,$$

   where $0 < \kappa_e < 1$ is the relaxation coefficient for the enrollment rate.
5. Using the decision rules obtained in step 3, $A^s(a,j)$ $\forall a, \forall j$, and Eq. (17), compute the age-dependent distributions by forward recursion. Then use these distributions $\lambda^s(a,j)$ and the age shares η_j to compute the per capita (next period) capital stock K_1 as follows:

   $$K_1 = \left(\sum_j \sum_a \sum_s \eta_j \lambda^s(a,j) A^s(a,j) \right) \Big/ (1+n),$$

   where n is the growth rate of cohort size. Check the convergence criterion ($|K - K_1|/K \leq tol_K$) to see if it needs to stop. If not, use the relaxation method to update K

   $$K_2 = \kappa_K K_0 + (1-\kappa_K)K_1,$$

   where $0 < \kappa_K < 1$ is the relaxation coefficient for capital. Then set $K_0 = K_2$, $e_0 = e_2$, and go back to step 1. The iteration will stop once all errors fall into the tolerance ranges.
6. Compute aggregate consumption, investment, tuition expense, and output by using the decision rules, age-dependent distributions, and age shares

   $$C = \sum_{j=1}^{J} \sum_a \sum_s \eta_j \lambda^s(a,j) C^s(a,j),$$

   $$P = \sum_{j=1}^{4} \sum_a \eta_j \lambda^c(a,j) p_j,$$

   $$X = (n+\delta)K.$$

7. Finally, check if the market clearing condition given by Eq. (21) holds. If it does, stop.

Appendix C. Algorithm to compute the transition path

In this paper we follow Chen et al. (2006) (also see Conesa and Krueger, 1999) in computing a transition path from the initial pre-1951 steady state toward a final steady state. In this way, we view 1952–2000 as a part of the transition path. Defining notation, we use $t=1$ for 1951, $t=T$ for the final steady state, and $t=2,\ldots,T-1$ for the transitional period. We take the following steps in the computation.

1. Compute the pre-1951 initial steady state by following the method described in Appendix B. Save the distribution $\lambda^s(a,j), \forall a, \forall j, \forall s$ for later calculation.
2. Feed in the exogenous change in the growth rate of HSGs' cohort size $\{n_t\}_{t=1}^T$, transformed capital-specific technological change $\{B_t\}_{t=1}^T$, and the tuition payments $\{p_t\}_{t=1}^T$.
3. Compute the final steady state at $t=T$. Save the value function $V^s(a,j), \forall a, \forall j, \forall s$ for later calculation.
4. Take the initial and final steady state values of capital stock K and enrollment rate e, and use linear interpolation to guess the sequences of $\{K_t\}_{t=1}^T$ and $\{e_t\}_{t=1}^T$. This is the initial guess for the transition path computation.
5. Start from $T-1$, take the value function of the final steady state as the terminal values $V^s(a,j,T)$, solve the individual optimization problem by backward induction, and obtain the decision rules for all cohorts through the transition path.
6. Use the distribution of the pre-1951 steady state as the initial asset distribution, together with the decision rules collected from step 5, and calculate $\{\lambda^s(a,j,t)\}_{t=2}^T$ by forward recursion. Then use them to calculate new $\{K_t\}_{t=1}^T$ and $\{e_t\}_{t=1}^T$.
7. Compare the new sequences of endogenous variables $\{K_t\}_{t=1}^T$ and $\{e_t\}_{t=1}^T$ with the initial guess and iterate until convergence.

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